MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. XCI. SUPPLEMENTARY NUMBER. No. 9

Notes on the Constitution of the Stars. By Henry Norris Russell.

1. The problem of stellar constitution, like all physical problems, must be attacked by the study of simplified mathematical models. The discussion of such models, as Milne justly remarks, should be an exact deductive science ; the applicability of the results to the actual problem depends mainly on our success in devising models, which is a difficult imaginative art. Both these activities may well be included under the term " theoretical astrophysics," which Milne would apparently confine to the first.

By common agreement the effects of rotation are put aside for separate discussion, and it is assumed that the material can be treated as a continuous medium, approximately at least, in thermodynamic equilibrium (thus excluding gaseous nebulæ and the outer parts of stellar atmospheres). There is, of course, good observational evidence to justify these assumptions for typical stars. Even so, the problem is too difficult for general deductive treatment, for two main reasons.

2. *First.* The properties of a gaseous medium of given composition are still imperfectly known.

(*a*) The equation of state, including the effects of ionization, appears to be pretty well established until the gas becomes degenerate at great densities, and the changes which then happen are now being worked out.

(*b*) The law of opacity is less certain. An approximate functional form for a highly ionized perfect gas is widely accepted ; but the numerical constants involved are still in debate. We are in serious uncertainty what the opacity is in stellar atmospheres, and ignorant of its behaviour in a degenerate gas.

(*c*) As regards the law of generation of energy within the stars, we know practically nothing. There is good reason to believe that some sub-atomic process or processes liberate energy at the expense of loss

of mass, but there are the widest differences of opinion about it—some suggesting that the rate of liberation increases rapidly with temperature and others that it diminishes. Until some approximate theory of energy-liberating processes can be derived from general physical principles, a true deductive theory of stellar constitution is out of reach. The situation is not hopeless. Milne * reports work in progress on the reversible transformation of matter into energy at temperatures of the order of 10^{11} degrees, and Atkinson † upon the building of heavy atoms from hydrogen at about $10^{7.5}$ degrees. But it may be long before this difficult question is cleared up.

3. *Second.* Even if we could calculate all the properties of a medium of specified composition, over an indefinite range of temperature and density, and with any desired accuracy, a fundamental difficulty would remain. The composition of the material involves the relative abundance of two or three hundred atomic species (including isotopes) all of which may behave differently as regards energy generation. A mere specification of the distribution of matter within a star involves therefore not a single density-function, but about three hundred such functions, one for atoms of each kind, dependent perhaps on one another, but in a way of which we have no idea.

It has, moreover, been suggested ‡ that, in the interior of the stars, there may be atoms heavier than any known on earth, which contribute importantly to the energy-liberation.

We can learn a great deal about the composition of a star's atmosphere, despite serious gaps due to the limitations of the observable part of the spectrum. But the atmosphere contains an exceedingly small fraction of a star's mass (about 10^{-11} in the case of the sun), and it may not be a fair sample of the composition of the whole. There are various diffusional processes which tend to concentrate heavy atoms in one part of a star (inside or outside, according to circumstances) and light atoms elsewhere, but these are so slow that it is doubtful whether they become important during a star's lifetime.§ On the other hand, there may be mixing by currents set in motion by the star's rotation ‖ or by recombination of electrons and ions.¶ Moreover, the energy-liberating processes may lead to the gradual exhaustion of some constituents, or to the building up of others. Finally, the question of the "initial" composition of the material, which may or may not have been the same in all parts of the mass when it began its existence as a star, remains unanswerable.

Here, in the writer's judgment, appears the great obstacle to the development of a complete deductive theory of stellar constitution. The number of variables involved, and the complicated way in which they may vary with position inside the mass, and with the time, seems to preclude all hope of an exact solution. It is necessary to work with

* *M.N.*, **90**, 778, 1930. † *Ap. J.*, **73**, 250, 1931.
‡ Jeans, *M.N.*, **86**, 561, 1926.
§ Eddington, *The Internal Constitution of the Stars* (*I.C.S.*), pp. 272–280.
‖ *Ibid.*, pp. 280–287.
¶ Unsöld, *Zeitschrift für Astrophysik*, **1**, 138, 1930.

drastically simplified models, and to test the appropriateness of the hypotheses upon which these are designed, by comparison with the observed facts. Two main lines of attack are open. We may adopt some particular equations of state, opacity, and energy-generation, and work out in detail the properties of the resulting stars ; or we may postulate some more general relation—such as homology of internal structure or identity of chemical composition—and study the properties common to all stars which have this distinctive feature.

4. Much of the work which has been done in theoretical astrophysics is of the first sort—for example, Eddington's original model * ($\kappa\eta$ = const.) and later point-source model,† and the recent investigations of Milne and his collaborators. In this case the " model " on which a star is built must be determined by numerical quadratures, and Emden's computations ‡ for polytropic spheres have had a great influence upon all later work. Such studies have the advantage that they lead to numerical results which can be compared directly with observation. They suffer, however, from a serious limitation. In order to obtain analytically tractable expressions, the underlying functional relations must be assumed to be very simple in form. There is a real danger that accuracy in the representation of the physical properties of the medium may be sacrificed to mathematical convenience ; and vigilance is necessary lest some conclusion, resulting solely from the analytical conditions which have been imposed, may be interpreted as a physical property of the stars.

For example, the simplest (and also the most severely restricted) assumptions which can be made are that, inside a given star, the molecular weight μ, the opacity κ, and the rate of generation of energy per unit mass ϵ are constants. But these assumptions impose upon the stars " weird and wonderful properties."§ They compel the conclusion that a sphere of perfect gas of given mass M can be in equilibrium (with finite central density) only if its luminosity has a certain definite value L_1 given by Eddington's equations $L_1 = M(1-\beta)/\kappa$; $\dfrac{1-\beta}{\beta^4 M^2 \mu^4} = \text{const.}$ If for such a mass $\epsilon = L_1/M$ the star can be in equilibrium with any radius, and is a polytrope of Emden type with $n = 3$. If, in such a body, ϵ should be slightly diminished, the star would lose heat faster than the internal sources supplied it, and contract—presumably until the gas laws no longer held ; while if ϵ should be increased, the star would expand (but be further than before from the theoretical configuration of equilibrium with $L > L_1$, in which the density becomes logarithmically infinite at the centre). The " diffuse " configuration is therefore, in this instance, unstable with regard to small changes in ϵ.

But if, instead of κ = const., we adopt the physically more probable law $\kappa = \kappa_0 \cdot \rho T^{-(3+n)}$, Jeans has shown || that, for small masses ($\beta \sim 1$)

* *M.N.*, **77**, 16, 1916. † *Ibid.*, **85**, 408, 1925.
‡ *Gaskugeln*, Leipzig, 1907.
§ Jeans, *Astronomy and Cosmogony* (1928), p. 82.
|| *M.N.*, **85**, 203, 1925.

there are polytropic perfect-gas configurations with index $3 + \frac{1}{2}n$. These are homologous and their luminosity follows the law $L = \text{const.}\ M(1-\beta)r^{-n}$. A star for which ϵ is slightly altered can now find a new configuration of equilibrium, with appropriate L, by a small alteration of its radius r. If n is negative, the initial shift will be toward the new configuration of equilibrium and the gaseous star will be stable ; * if n is positive, it will be unstable.

For larger masses the configurations with the same M but different L are not polytropic but are homologous, and L still varies as r^{-n}.

In general, for stars in which ϵ is fixed but κ (and perhaps also μ) varies with the pressure and temperature according to definite laws, there will be configurations of equilibrium with the same mass but different radii, each of which has its appropriate value of L and hence of ϵ. These configurations will be stable with respect to changes in ϵ provided that $\dfrac{\partial L}{\partial r}$ is positive.

All this is an old story ; but it is worth repeating, in view of recent suggestions that, because perfect-gas models with constant κ are unstable, perfectly gaseous stars must be unstable.

5. In the writer's opinion it is to be regretted that so much labour is being spent on calculations based on the postulates $\mu = \text{const.}$, $\kappa = \text{const.}$, $\epsilon = \text{const.}$; for there is good evidence that all three of these assumptions are physically incorrect.

Deep inside a star, where ionization is great, the values of μ must be nearly the same for the heavier atoms ; † but they are much lower for hydrogen and helium, and the general mean must depend on the abundance of the latter. Near the surface μ increases rapidly, except for *H* and *He* (which are neutral only in a negligibly thin surface layer). The region of diminished ionization for other elements probably includes but a small part of the star's mass. If it has a correspondingly small influence on the luminosity, it may be a good approximation to treat μ as constant inside a given star, but it will not be the same from star to star if the proportion of hydrogen varies seriously.

There are strong theoretical reasons for believing that κ varies approximately as $PT^{-\frac{3}{2}}$ (P being the electron-pressure), and Milne ‡ has collected much observational evidence in favour of such a law of variation, at least in the photospheric layers. The constant of proportionality varies with the composition of the gas.

Finally, there is direct observational evidence that the mean value of ϵ varies enormously from star to star and is practically zero in the earth and the other planets. There can be no thought of regarding ϵ as a constant of nature. It must vary in some way with circumstances ; and probably no one supposes that it is actually constant throughout the whole of a star. Most investigators, though not all, consider it likely that it is greater in the hot interior than near the surface. If this is so, the assumption $\epsilon = \text{const.}$, besides its analytical convenience, is valuable, like the point-source model, as a limiting case.

* Russell (*P.A.S.P.*, **31**, 205, 1919), or *Nature*, **116**, 209, 1925.
† Fowler and Guggenheim, *M.N.*, **85**, 939, 1925. ‡ *M.N.*, **89**, 174, 1928.

6. It appears to be of very doubtful advantage to assume, as Milne does,* that the luminosity of a star, as well as its mass, is determined in advance by the presence within it of sources of fixed total output. Such a body would indeed settle down into a steady state, if this were possible; but the resulting configuration of equilibrium might be quite different from that of an equal mass in which ϵ, κ, and μ varied with temperature and pressure, although the luminosities were identical. The range of luminosity within which equilibrium is possible with a given mass may be different in the two cases, and, above all, the equilibrium may be stable with one law of energy-generation and unstable with another, even though the luminosity, the mass, the law of distribution of density, and the rate of generation of energy at each point in the particular configuration of equilibrium under consideration are all the same.

It is obviously unsound to conclude that, because stars having a given property (such as a low mean density) exist in nature, there must therefore be stable solutions of some theoretical problem which has been artificially simplified by severe limitations (*e.g.* constant ϵ and κ) which possess the same property. If there are none, this means only that these limitations do not always (if ever) occur in nature.

It is well worth while to determine what changes in the theoretical postulates (*e.g.* the introduction of an incompressible core) will permit the existence of stable solutions having the desired property; but it does not follow that the stars actually possess other properties of these solutions (such as a very high central density). It has at best been proved that they *may* possess them. More cannot be said until the theoretical possibilities of other postulates have been exhausted—or until new observational evidence is brought to bear.

7. Of the studies based on functional form, the best known deal with homologous stars. Here the consequences of an assumed law of density, with assigned expressions for κ and μ, are worked out. In this case the value of ϵ (though not its general functional dependence on pressure and temperature) may be calculated for all points within the star. For ill-chosen models ϵ may come out negative through considerable regions.

The resulting general formula connecting the mass inside any concentric sphere and the flux of energy outward through it has been given by Vogt,† and three years earlier by the writer ‡—who, however, not only buried it in a textbook but camouflaged its grave by "avoiding the use of the calculus." With the assumed law of opacity $L \propto r^{-\frac{1}{2}}$, and there is no rigid mass-luminosity relation; but the luminosities may be reduced to equal radii or equal effective temperatures by the application of corrections which are small for all but the extremes of stellar temperature, and a definite mass-luminosity curve is then obtained.§

* *M.N.*, **91**, 7, 1930. † *Veröff. Jena*, **3**, 9, 1930.
‡ Russell, Dugan, and Stewart, *Astronomy* (Boston, 1927), p. 899.
§ Eddington, *M.N.*, **84**, 308, 1924.

The form of this curve is remarkably insensitive to changes in the model on which the stars are assumed to be built. Its zero-point (the luminosity of a star of given mass and effective temperature) varies somewhat with the model, but more with the opacity-coefficient, and most of all with the molecular weight.

The close agreement between the observed curve and that resulting from Eddington's model is therefore no evidence in favour of this particular model,* or of more than a roughly approximate homology of structure for different stars. It appears rather to indicate either that the molecular weight is nearly the same in corresponding parts of the stars at large, or that differences in the model systematically counteract those in molecular weight.

A varying admixture of hydrogen would change μ and κ, and produce great differences in the luminosity of homologous stars of the same mass.† Hydrogen is so abundant in the atmospheres of the stars that one might expect it to be abundant all through them. If present in suitable proportion it might remove the serious discrepancy between the "physical" and "astronomical" values of the absorption coefficient. But this proportion varies with the mass of the star, and it is not easy to see why the actual stars should have their hydrogen content adjusted in this way. It should be remembered, however, that almost all the stars for which M and L are known observationally with any approach to accuracy belong to the main sequence. Of those which do not, Capella, the only representative of the giants, agrees with the rest. The white dwarfs are seriously discordant, confirming the view that they are not homologous with other stars.

8. It may be appropriate at this point to refer to a closely related matter, on which the writer is unable to agree with Professor Milne—namely, that a certain "discriminant" quantity C (whose value could be determined from observable data if the law of opacity in the photosphere were known) "recapitulates in itself those aspects of the density distribution which are significant for the surface cooling: it is a telescoped version of the structure of the star between centre and boundary." ‡ Milne reaches this conclusion as follows:—

If P_c, ρ_c are the central pressure and density, and P, ρ those at distance r, he sets

$$P = P_c y, \qquad \rho = \rho_c z, \qquad r = \left(\frac{P_c}{4\pi G {\rho_c}^2}\right)^{\frac{1}{2}} x \quad . \qquad (M\ 4, M\ 6)\ \S$$

Then, setting

$$z = \theta^3, \qquad y = \theta^4 \phi(\theta) \quad . \qquad . \qquad (M\ 18, M\ 19)$$

he shows that θ, ϕ satisfy the differential equation

$$\frac{4}{x^2}\frac{d}{dx}\left[x^2 \frac{d\theta}{dx}\left\{\phi(\theta) + \tfrac{1}{4}\theta \frac{d\phi}{d\theta}\right\}\right] = -\theta^3 \qquad . \qquad (M\ 20)$$

---

* Russell, *op. cit.*, p. 893; Vogt, *op. cit.*, p. 17.
† Eddington, *I.C.S.*, p. 244.
‡ *M.N.*, **90**, 51, 1929.
§ These are the numbers of Milne's equations; *M.N.*, **90**, 26–29, 1929.

At the surface ($x = x_1$) $\theta \to 0$. If $\phi \to \phi_1$ (a finite value) the mass is given by

$$M^2 = \frac{4P_c^3}{\pi G^3 \rho_c^4} x_1^4 \phi_1^2 \left(\frac{d\theta}{dx}\right)_1^2 \quad . \quad . \quad (M\ 22)$$

If the outer layers are gaseous, the ratio of radiation-pressure to total pressure is given by

$$\frac{1-\beta}{\frac{1}{3}a\beta^4}\left(\frac{R}{\mu}\right)^4 = \frac{P^3}{\rho^4} = \frac{P_c^3}{\rho_c^4}\phi^3 \quad . \quad . \quad (M\ 23)$$

Hence for a point just inside the photosphere

$$\frac{1-\beta_1}{\beta_1^4}\left(\frac{R}{\mu}\right)^4 = \frac{\pi a G^3 M^2}{12} \frac{\phi_1}{x_1^4 \left(\frac{d\theta}{dx}\right)_1^2} \quad . \quad . \quad (M\ 24)$$

The last factor is Milne's C. He argues " since $\theta(x)$ is the solution of a differential equation containing ϕ, $(d\theta/dx)_1$ depends on the complete march of ϕ from centre to photosphere, and hence so does $1 - \beta$. Thus $1 - \beta_1$ depends on the complete density-distribution and not merely on the local value of ϕ." Now the quantities P_c, ρ_c which Milne employs as units for y, z are essentially " unobservable." In their place we may substitute the " observable " quantities P_0, ρ_0, defined as the central density and pressure of a homogeneous sphere of the same mass and radius as the star.* These are given by the equations

$$\rho_0 = \frac{3}{4\pi}\frac{M}{r_1^3} \qquad P_0 = \frac{3G}{8\pi}\frac{M^2}{r_1^4}$$

If P_0, ρ_0 are employed as units, the analysis proceeds exactly as before, but the numerical coefficients are greatly simplified. Setting $P = P_0 y'$, $\rho = \rho_0 z'$, etc., equation (M 6) becomes

$$r = x' r_1 \sqrt{\tfrac{1}{6}} \quad . \quad . \quad . \quad . \quad (R\ 6)$$

Also $P_0^3/\rho_0^4 = \frac{1}{6}\pi G^3 M^2$, so that (M 22) becomes

$$x'_1{}^4 \phi'_1{}^2 \left(\frac{d\theta'}{dx'}\right)_1^2 = \frac{3}{2} \quad . \quad . \quad . \quad (R\ 22)$$

and (M 23) and (M 24) both reduce to

$$\frac{1-\beta_1}{\beta_1^4}\left(\frac{R}{\mu}\right)^4 = \frac{\pi a G^3}{18} M^2 \phi'_1{}^3 = \frac{\pi a G^3}{18} M^2 \frac{y'_1{}^3}{z'_1{}^4} \quad . \quad (R\ 23)$$

The last equation is identical (except for notation) with one given by the writer four years ago.† The preceding equation expresses a restriction placed upon the solutions of the differential equation (M 20) near the boundary. This is imposed by the condition prescribed by Milne, that ϕ shall tend to a finite limit. If this condition (which, as Eddington

* Russell, Dugan, and Stewart, *Astronomy*, p. 896.
† *Ibid.*, p. 897, equation (16).

has pointed out,* is not always justified) is removed, $\phi_1 + \frac{1}{4}\theta_1\left(\frac{d\phi}{d\theta}\right)_1$ must be substituted for ϕ_1 in (M 22) and (R 22), while (M 23) and (R 23) remain unaltered, and Milne's discriminant becomes

$$C = \frac{{\phi_1}^3}{{x_1}^4\left(\frac{d\phi}{d\theta}\right)_1\left(\phi_1 + \frac{1}{4}\theta_1\left(\frac{d\phi}{d\theta}\right)_1\right)^2}$$

From (M 23) and (M 24) we have, in all cases

$$C = \frac{4P^3}{\pi G^3 M^2 \rho^4}$$

so that its value at a given point in a given star depends only on the star's mass and the local pressure and density. If the latter are expressed in terms of the observable units P_0 and ρ_0 we get the simple equation

$$C = \frac{2}{3}\frac{{y'_1}^3}{{z'_1}^4} = \frac{2}{3}{\phi'_1}^3$$

while with Milne's notation

$$C = \frac{4{P_c}^3}{\pi G^3 M^2 {\rho_c}^4}{\phi_1}^3$$

The apparent dependence of C upon "the complete march of ϕ from centre to boundary" therefore comes only from the choice of unknown instead of known units of measurement.

As Milne has pointed out† there is good reason to believe that P^3/ρ^4 is nearly constant in the photospheric region of a star; but all that C tells us is the ratio of this constant to M^2. It gives some information about the outside of a star, but none at all about the inside that we did not have already. There is therefore no justification for the assumption that stars for which C is the same are similar in internal constitution. Provided, of course, that we do not have or postulate knowledge of the full behaviour of μ, κ, and ϵ as functions of P and ρ. If such postulates are made (as in Milne's theory), C determines the internal constitution. On this point the writer is in full agreement with Eddington.

9. A still more general postulate of the functional type is that we have to deal with stars of fixed and definitely known composition. In this case the writer,‡ and Vogt independently,§ have stated the theorem —which is not as well known as it should be—that stars of fixed composition must exhibit, not only an exact relation connecting mass and luminosity, but similar exact relations between mass and radius, luminosity and effective temperature, and any other pair of their macroscopic properties.

* *M.N.*, **90**, 284, 1930. † *Ibid.*, **90**, 28, 1929.
‡ Russell, Dugan, and Stewart, *Astronomy*, p. 910, 1927.
§ *Veröff. Jena*, **3**, 19, 1930.

The proof is simple. The equations for the internal equilibrium of a star may be written

$$\left.\begin{aligned} d\mathrm{M}(r) &= 4\pi\rho r^2 dr \quad (1) & d\mathrm{L}(r) &= 4\pi\epsilon\rho r^2 dr \quad (2) \\ g &= \mathrm{G}\mathrm{M}(r)/r^2 \quad (3) & \mathrm{H} &= \mathrm{L}(r)/4\pi r^2 \quad (4) \\ d\mathrm{P} &= -g\rho dr \quad (5) & dq &= -\frac{\mathrm{H}\kappa}{c}\rho dr \quad (6) \\ \mathrm{P} &= p+q \quad (7) & q &= \tfrac{1}{3}a\mathrm{T}^4 \quad (8) \end{aligned}\right\} (\mathrm{A})$$

where p, q, and P are the gas-, radiation-, and gravitational-pressures, and H the energy flux per unit area.

We now make two postulates : *first*, that ρ, κ, and ϵ depend only on the local values of p and T (or of p and q) and *second*, that they are everywhere finite. No assumptions are made regarding the forms of ρ, κ, ϵ as functions of p, T, except that they are *definite* (involving no arbitrarily disposable parameters), *finite*, and *one-valued*. They need not be continuous.

The equations (A) then form a system of the fourth order in the variables p, q, and r. Starting with any chosen value of r, we may assign arbitrary values to p, q, determining ρ, κ, ϵ, and to g and H, fixing the gradients of p and q. The solution may then be continued inward and outward by quadratures, and is uniquely determined. To represent a physically possible star, our solution must satisfy certain boundary conditions. At the surface p and q must vanish together. More precisely p approaches zero and q a finite value q_0, depending on the flux at the surface, which is however so small in comparison with the values in the interior that, except close to the surface, the difference between the solutions for which $q \to 0$ and $q \to q_0$ can be neglected.* At the centre there are two boundary conditions—that there shall be no point-mass or point-source, positive or negative—or, more specifically, considering that ρ and ϵ must be finite, that g and H shall be of the order of r, and L and M of r^3, when r is small.

These three conditions leave but one degree of freedom for the solutions of (A) which satisfy them, so that the latter must involve but a single disposable constant. The smallest positive value of r for which P vanishes—that is, the radius of the star—will be a function of this parameter and so will the corresponding values of M, L, H, T_e, and so on. If any one of these quantities is plotted against any other, the resulting points will lie on a definite curve, expressing a relation between the two quantities which is a necessary consequence of our assumptions. These curves may be of complex form, and there may be more than one possible star with a given mass, but there will not be a range of values of one variable corresponding to a fixed value of the other, unless the functional relations happen to be such that one of the variables disappears from the final equations. This happens, for example, in the perfect-gas case with ϵ, κ, μ constants. The curves connecting M and r, L and r reduce to the straight lines M = const., L = const.,

* Jeans, *M.N.*, **85**, 202, 1925.

while that on the M, L diagram shrinks into a single point. Such cases of functional degeneracy will be exceptional.

10. We may pause here, and consider what restrictions have really been imposed in deriving this result. Equation (7) assumes that no other forces than the pressure of the medium and of radiation are effective in sustaining the mass against gravity, and also, as Milne has pointed out,* that the stresses caused by the flow of radiation reduce to a pressure—which should be the case for the spherical distributions here considered. Equation (6) assumes that the flux of energy at any point depends only on the local conditions, including the temperature gradient. With a suitable definition of κ, this may be made to take account of conduction as well as radiation, but not of convection—which must therefore be supposed to be of negligible importance. Finally the assumption that, in (2), ϵ depends only on the local values of p and T, rules out heat produced by gravitational contraction—for, in the general case, where the law of internal distribution of density is changing as the star contracts, we cannot even fix the sign of the work done by gravity on a given element of the mass until we know the law of this change.

The basal assumption that ρ, κ, and ϵ are definite functions of p and T should be valid either if the relative proportions of all atomic species are constant throughout all parts of all the masses considered, or if these proportions vary in such a way that, at a fixed temperature and pressure, the composition is always the same. The latter situation might be brought about, inside a single star, by processes such as diffusion ; but it is unlikely that such processes would always result in the same composition for parts of different stars at the same p and T, even though the mean composition of the stars were the same. Identity of local composition could be brought about by a very slow internal circulation—sufficient to counteract the effects of diffusion and of possible exhaustion of active material, but not enough to vitiate the assumption of hydrostatic equilibrium, as an adequate approximation. Such a circulation might be brought about by a slow rotation † and the assumption of uniform mixing appears therefore to be physically reasonable.

It is further necessary to assume that, throughout the mass, there is no separation of the material into phases of different composition and density at the same pressure and temperature, such as would happen when the most refractory constituents began to liquefy, for the two phases would then segregate under gravitation and destroy the uniformity of composition. This excludes the later stages of cooling bodies, but is probably far from affecting any known star.

A change of state affecting the whole medium, without change of composition, such as the condensation of a vapour into a liquid, is permissible, since in this case the density is a one-valued function of temperature and pressure, though discontinuous at the boiling-point. In view of the known complexity of composition of the stars, this case

* *M.N.*, **91**, 755, 1931.
† Eddington, *M.N.*, **90**, 54, 1929.

is not likely to occur unless, as Milne suggests,* it may happen at the onset of the limiting condition of incompressibility at the maximum possible density of matter. That there must be finite limits for ρ, κ, and ϵ can hardly be questioned.

These various limitations, although considerable, are far less restrictive than those which are usually imposed in discussions of the problem. Ionization and chemical combination have full play—so long as no reaction-products condense out—and the equation of state may include the effects of electrostatic attraction, degeneracy (it may be in successive stages), and possible " jamming " of partially ionized atoms, as suggested by Jeans. The laws of opacity and energy-generation may be equally general.

11. It follows that any investigation which starts from the postulate, or ends with the conclusion, that a star of given mass may have any assigned luminosity within given limits, must involve, explicitly or tacitly, the assumption that the stars differ in composition. The most obvious illustration is the assumption that ϵ is a constant, independent of physical conditions within the star. If the last three words are deleted, it follows that for all stars of the assigned composition, $L = \epsilon M$, giving a definite mass-luminosity law (and one inconsistent with observation). If, for instance, we take Milne's " collapsed " stars in which an inner core is degenerate and the outer part a perfect gas, while ϵ, κ, and μ are constants, a star of given mass M may have any luminosity less than L_1 according to the size of the core ; but no two such stars have the same value of L/M, or of ϵ, and they are all of different composition. If, however, from the various collapsed configurations of different masses, we pick out those for which $L/M = \epsilon_0$, we shall have a series of stars of fixed composition but differing in mass, luminosity, radius, and effective temperature, and forming a one-parameter family. If complete degeneracy set in abruptly, this series would end abruptly at the point where the condition for degeneracy was just satisfied at the centre, to be continued by " diffuse " stars built on Eddington's model and all of the same mass and luminosity—the whole series forming a one-parameter family as regards the radius or the mean density. If the change in the equation of state is gradual, there will be a gradual transition from one series to the other, but still a one-parameter family.

12. An instructive example is furnished by the point-source model, in which it is assumed that $\epsilon = 0$ and all the energy comes from a miraculous central point. Here L is to be set constant in equations (A), which reduce to the third order. There are now two boundary conditions—the absence of a central point-mass, and the same one as before at the surface—and, for a prearranged value of L, we will find a one-parameter family of stars of fixed composition, but, in general, with different masses and radii. If L is now allowed to change from star to star we have no mass-luminosity relation at all, or at best one which confines the luminosity corresponding to a given mass to a continuous range of greater or less extent. This seems at first sight to be inconsistent

* *M.N.*, **91**, 480, 1931.

with the theorem regarding stars of fixed composition. But to specify κ and μ (as functions of p and T), and then allow L to take any arbitrary value is clearly comparable, not to specifying κ, μ, and ϵ exactly, but to specifying κ and μ and letting ϵ vary from star to star with one degree of freedom (as for example by an arbitrary factor). In both cases our physical specifications are not definite, but contain a disposable constant. This, with the additional degree of freedom coming from the solution of the differential equations, under the boundary conditions, gives us a two-parameter family of stars, and abolishes the mass-luminosity relation.

These conclusions are confirmed by Cowling's study * of the point-source model of perfect gas for which κ and μ are constants. He finds that for a given L there is a singly-infinite series of homologous stars, with no central point-mass and with the appropriate boundary condition at the surface. His equations (9) and (10) show that from any solution $T = f(r)$ others may be obtained by setting $T_1 = xT$, $r_1 = r/\theta x$ where x is any constant and $\theta^3 = \mu/\kappa L \times$ abs. const. Then, at homologous points, by his equation (2), $p' \propto x^4$; by (3) $\rho \propto x^3/\kappa L\theta$. But $M \propto \rho r^3 \propto 1/\kappa L\theta^4 \propto (\kappa L)^{\frac{1}{4}}\mu^{-\frac{1}{4}}$ so that $L \propto M^3\mu^4/\kappa$.

When μ and κ are fixed, models of a given mass may have any radius, but are of a fixed luminosity. For other luminosities there is a positive or negative point-mass.

From the physical standpoint, when the point-source model is regarded as an approximation to an actual case of stellar equilibrium, to be "finished off" near the centre by a transition to other laws of energy-generation and density, Mr. Cowling is justified in saying that "no models with a positive central point-mass may be rejected. Thus any mass in a certain range is consistent with the given luminosity." But this change in the properties of the material near the centre is not explicitly defined, and it evidently introduces at least one new disposable constant into the problem, and so is inconsistent with the limitation to stars of fixed composition. Cowling's model applies to any star for which $\kappa L(r)$ is constant (like Eddington's when $\kappa\eta$ = const.). In both cases stars of different masses are homologous; but this is not usually true, though there are always homologous solutions for stars of the same mass.

If r_1, ρ_1, etc., correspond to any particular solution of (A) and we set $r = ar_1$, $\rho = b\rho_1$, $T = cT_1$, $\mu = d\mu_1$, we have $M(r) = a^3bM_1(r_1)$, $P = a^2b^2P_1$, $q = c^4q_1$ and, by the gas-laws, $p = bcp_1/d$. If (7) is to be satisfied we must have $a^2b^2 = c^4 = bc/d$, whence $a^3bd^2 = 1$, or $M\mu^2$ = const.—the necessary condition for homology. For homologous stars of the same mass, L is constant when κ is constant, but, if $\kappa \propto \rho/\mu T^{3+n}$, $L \propto r^{-n}$.

The point-source model with $n = \frac{1}{2}$ has been worked out by Biermann,† who reports that the relation between M and L is in close agreement with observation, and that the discordance between the physical and astronomical values of κ disappears. Full details of this important work will be of great interest.

* *M.N.*, **91**, 92, 1930. † *Zs. f. Astrophysik*, **2**, 244, 1931.

13. The simplest case of variable composition is that in which the properties of the various atomic species depend on one adjustable parameter x, constant throughout the star, and, it may be, on p and T as well. All stars for which x has the same value x_0 will form a one-parameter family of the kind already discussed. For each value of x_0 there will be a definite mass-luminosity curve; but as x_0 changes we will have a one-parameter family of curves, one or more of which will pass through every point in the LM plane (though in some parts of it the solutions may not be real).

In this case there will in general be no recognizable mass-luminosity relation. But if the curves have an envelope it may be that, over a certain range of values of L, M, and for values of x_0 within limits, the curves lie fairly near the envelope, and an approximate mass-luminosity relation will exist. The same may happen if the functional forms of ϵ, κ, ρ are such that the curves lie close together, without crossing, over a part of their course. In either case, the curves may lie close together on one diagram—say that connecting mass and luminosity—and not on another, *e.g.* for luminosity and effective temperature.

When the composition depends on two or more free parameters, only some peculiarity in the form of the functions defining ϵ, κ, and μ can bring the doubly-infinite set of possible curves close enough together to produce any approximation to a mass-luminosity relation.

14. The observational data show very much the characteristics which have just been described. On a mass-luminosity diagram most of the points lie so near a smooth curve that there are few instances in which the discrepancy cannot be ascribed to errors of observation. There are some real exceptions, however. The white dwarfs are all much too faint for their masses. Other undoubted exceptions are found among eclipsing binaries such as RS CVn.* and RT Lacertæ.† In the latter system the well-determined masses are 1·9 and 1·0 times the Sun's and the difference of brightness should be about two magnitudes. Luizet's estimates ‡ indicate a difference of $0^m{\cdot}4$, and unpublished photometric work by Mr. Merrill at this observatory a much smaller difference. Both stars are of low mean density.

On the well-known diagram connecting luminosity and effective temperature (or spectral type) the points are much more widely scattered. Along the main series there is a strong concentration near a mean curve, but the scattering is too great to be attributed wholly to accidental error. The white dwarfs seem to follow a roughly parallel curve,§ though the scanty data indicate a wide dispersion. But the giant and super-giant stars are scattered so widely that there is obviously no hope of fitting all the points by a single curve.

Now if the stars were uniform in composition, no matter what might be the properties of the material composing them, the points on *both* diagrams should lie along definite curves. It may therefore be concluded, with considerable security, that the stars actually differ

* Joy, *Ap. J.*, **72**, 45, 1930. † Joy (in press).
‡ Miss Fowler, *Ap. J.*, **52**, 257, 1920.
§ Russell and Atkinson, *Nature*, **127**, 661, 1931.

in composition—atoms of various kinds being present in different proportions, either in the stars as a whole or from part to part of them. There is nothing new about this conclusion. Direct spectroscopic evidence in its favour is found in the branching of the spectral sequence into classes M, N, and S, and the hypothesis of exhaustion of "active" matter has been made in almost all theories of stellar evolution. But evidence of a new type may be of value.

15. The practical problem is to explain why there is such a good empirical mass-luminosity relation, despite the wide dispersion in the relation between luminosity and colour. The principal reason is probably that already mentioned (§ 7)—that the luminosity is relatively insensitive to changes in the model on which the stars are built; but the form of the opacity-function also has a share. With the generally accepted law, the luminosity of stars of the same mass should vary as $r^{-\frac{1}{2}}$ or as T_c^{4}. The great majority of the stars have effective temperatures between 3000° and 12,000°, and for these the correction to reduce the absolute magnitude to a standard temperature of 6000° never exceeds $\pm 0^{m}\cdot 60$. There are very few stars for which it is greater than one magnitude. This refers to the bolometric magnitude; when the correction to visual magnitude * is applied the luminosity for constant mass behaves as follows:—

T	2000°	3000°	3600°	6000°	9000°	12,000°	18,000°	30,000°
M_v	$+3\cdot0$	$+2\cdot0$	$+1\cdot1$	$0\cdot0$	$0\cdot0$	$+0\cdot3$	$+1\cdot1$	$+2\cdot4$

Over the whole spectral range from B2 to K5 the visual absolute magnitudes should therefore be within half a magnitude of a mean value. The deviations increase rapidly for temperatures below 3000° and above 25,000°. Observational selection will be heavily against the inclusion of such stars in any catalogue based on apparent magnitude—a familiar explanation of the fact that very few are found. If space were full of stars which, for each given mass, had indiscriminately all manner of radii, and hence of effective temperatures and luminosities, this selection effect alone would produce a marked correlation between mass and visual luminosity among catalogue stars, if no correction for effective temperature were applied. There would, however, be little or no correlation between luminosity and effective temperature.

The mass-luminosity relation appears, therefore, to be a very poor test for theories of stellar constitution. The relation between luminosity and spectral type affords a much severer criterion, and unfortunately one which no existing theory comes near satisfying.

The suggestion that ϵ is insensible until a temperature of the order of $10^{7\cdot5}$ degrees is reached, and increases rapidly † with increasing temperature (not abruptly—this is a misunderstanding) has at least the merit of leading to a luminosity-spectrum relation similar in character to the main sequence. By adding the postulate that the actual evolution of heat involves a considerable time-lag Eddington ‡ has

* Hertzsprung-Eddington, *I.C.S.*, p. 139.
† Russell, Dugan, and Stewart, *Astronomy*, p. 913.
‡ *I.C.S.*, p. 300.

removed the objection of pulsational over-stability; and Atkinson's work shows that both postulates may have a physical basis. But until such a theory can show why a star of the sun's mass should settle down with an effective temperature of 6000° rather than 2000° or 20,000° it is obviously very incomplete.

16. In conclusion, the writer desires to express his complete agreement with Milne's statement that the great obstacle to progress in the theory of stellar constitution is our ignorance of the process of energy liberation. So long as this continues the problem is indeterminate, with an uncertainty measured not by an unknown constant, but by an unknown function. It is earnestly to be hoped that investigations in atomic physics may be able to supply the missing link. Whenever a new relation can be established on physical principles, such as the equation of state of a degenerate gas, new and solidly-founded advances may be anticipated, as is illustrated by Milne's brilliant success in accounting for the general properties of the white dwarfs. Meanwhile, one may attempt to fill the gap by assuming some other functional relation among the variables; but these assumptions are necessarily more or less arbitrary.

If some such *ad hoc* hypothesis should succeed in representing the relation of luminosity to effective temperature as well as to mass, there would be ground for satisfaction, but not for certainty that the postulates corresponded to reality. It is possible that several different sets of assumptions might give results accordant with the observed data, within their rather wide uncertainty.

Independent information regarding the degree of central concentration of density may become available from the motion of periastron in eclipsing binaries. The interesting case of Y Cygni presents unexpected complications. A thorough study of all other probable cases of apsidal motion would abundantly repay the effort, and is greatly to be desired.

The results of different theoretical approaches—when cleared of the slips that are inevitable in pioneer work—should therefore be regarded as supplementing one another, rather than as resembling Cleon's

> "three books on the soul,
> Proving absurd all written hitherto
> And putting us to ignorance again."

Summary.

17. (1) A complete deductive theory of stellar constitution is impracticable, on account of our present ignorance of the law of generation of energy, and, above all, because the atomic composition of the star involves about three hundred independent variables, in a complicated manner.

(2) The specific model which has been much discussed of late—in which ϵ, κ, and μ are assumed to be constant—suffers from two defects. Its postulates are almost certainly contrary to fact, and their functional

form is such as to impose peculiar properties upon the resulting " stars." Conclusions from comparisons of the actual stars with this model should therefore be received with caution.

(3) The success of the homologous model in representing the observed mass-luminosity relation depends mainly (as Vogt points out) on the fact that this relation is insensitive to considerable changes in the model. The mean molecular weight, however, appears to be nearly the same in all stars—or else changes in the model counteract its variations.

(4) Professor Milne's discriminant C depends upon conditions in the outer layers of a star, but involves no property of the interior except its total mass.

(5) Stars of fixed composition should exhibit exact relations between mass, luminosity, effective temperature, etc. The conditions under which this can be proved are discussed, and the conclusions illustrated, by means of the " collapsed " and " point source " models.

(6) The relation between luminosity and effective temperature gives a much more searching test of theories of constitution than that between mass and luminosity. The observed data indicate that the stars must differ in composition—probably to a considerable extent.

(7) Until some physical theory of the energy-generation process becomes available, all theories of stellar constitution must involve a speculative element. Different investigations should be regarded as supplementary rather than antagonistic.

In conclusion it is a pleasure to express the writer's deep indebtedness to Professor Milne, Professor Atkinson, and Mr. Joy, who have courteously communicated important papers in manuscript or in proof.

Princeton University Observatory :
1931 *June* 12.

The Contours of Emission Bands in Novæ and Wolf-Rayet Stars.

By C. S. Beals, M.A., D.I.C., Ph.D.

Introduction.

In previous publications * the present writer has advanced a theory to account for the broad band emission of Wolf-Rayet stars. This theory attributes the appearance of emission bands to the continuous ejection of atoms from the surface of a star by radiation pressure. The observational evidence so far advanced in support of this theory has had to do mainly with the variation of band width with wave-length and with the appearance of absorption borders on the violet edges of emission bands. Certain points of similarity between Wolf-Rayet spectra and the spectra of novæ (where the hypothesis of violent

* *M.N.*, **90**, 202, 1929; *Pub. D.A.O.*, **4**, No. 17, 1930.

ejection of gases seems reasonably well established) have also been considered as evidence favouring the theory.

The present paper contains a discussion of the theory in its relation to band contours. On the basis of the ejection theory, it can be shown that, if certain reasonable assumptions be made concerning the mechanism of emission, it is possible to make predictions concerning the forms of emission band contours, which can be tested observationally. These predictions, which apply equally to novæ and Wolf-Rayet stars, are discussed in connection with spectro-photometric results on band contours obtained at this observatory. In developing the mathematical theory of band contours the writer has had the benefit of advice and criticism from Dr. R. O. Redman, whose assistance is gratefully acknowledged.

Theory of Band Contours.

If the assumptions be made (*a*) that radiation from each atom ejected contributes equally to the intensity of the emission band, and

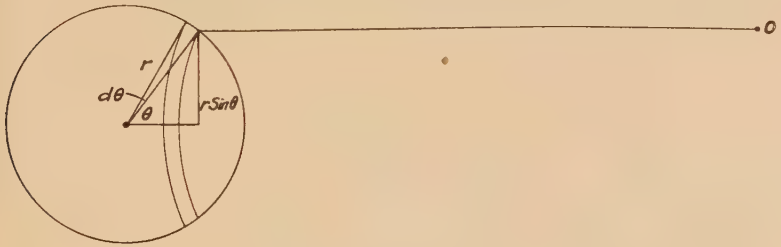

Fig. 1.

(*b*) that all atoms are ejected with the same velocity, we may derive an expression for the band contours as follows (see fig. 1) :—

Consider a shell of radius r surrounding the star. Let n be the number of atoms ejected per second from unit area of this shell. Let v be the velocity of ejection. If, now, we take a zone of radius $r \sin \theta$ and width $d\theta$, symmetrical with respect to the line of sight, the area of the zone will be $2\pi r^2 \sin \theta d\theta$; the number of atoms passing through it per second will be

$$N = 2\pi n r^2 \sin \theta d\theta,$$

and the energy which they will contribute to the band will be

$$dE = K \cdot 2\pi n r^2 \sin \theta d\theta \quad . \quad . \quad . \quad (1)$$

where K is some undetermined constant.

If, now, we denote the radial velocity for an observer at O as U, then

$$U = -v \cos \theta.$$

Differentiating this expression and solving for $d\theta$ we get

$$d\theta = \frac{dU}{v \sin \theta} \quad . \quad . \quad . \quad . \quad (2)$$

Substituting this value of $d\theta$ in (1) gives

$$dE = \frac{\text{const.}}{v} dU \quad . \quad . \quad . \quad . \quad (3)$$

which represents the energy in the velocity range from U to U + dU.

Equation (3) may also be written in the form

$$dE = \frac{\text{const.}}{v} d\lambda \quad . \quad . \quad . \quad . \quad (4)$$

since $dU = \frac{c}{\lambda_0} d\lambda$ from Doppler's principle.

Since dU may be treated as a constant, dE is also a constant and the contour will be perfectly flat on the top, will have a total width of $2v$, and will have perpendicular sides. This theory applies to any case of the ejection of atoms from a stellar surface in which the above conditions are fulfilled whether by a single outburst or by a continuous process.

A previous attempt by Menzel * to derive an expression for the band contour on the basis of similar assumptions led to an expression

$$I = I_0 \sin \theta$$

or

$$I = I_0 \sqrt{1 - \frac{c^2}{{v_0}^2}\left(\frac{\lambda_0}{\lambda} - 1\right)^2}.$$

Menzel apparently made the error of treating the differential coefficient $d\theta$ in (1) as a constant. Actually $d\theta = \frac{dU}{v \sin \theta}$ as in (2).

It is scarcely to be expected that under the actual conditions prevailing in the neighbourhood of novæ or Wolf-Rayet stars the assumptions on which equation (3) was derived will hold precisely. The first assumption, that each atom contributes equally to the intensity of the band would appear, to a first approximation, to be a reasonable one, and in order to avoid complications it will be retained. It is extremely unlikely, however, that all velocities of ejection will be equal, and the real problem is therefore to be able to compute the contour of a band produced by a known distribution of velocities in which there is a considerable spread. The problem is actually a simple one and may be worked out by a series of approximations, in which the frequency curve of velocities is divided into narrow strips of constant width, the velocity for each strip being taken as constant and equal to its mean value over the small section of the curve concerned. Each strip can then be treated separately on the basis of equation (3) stated in the form

$$dE = \text{const.} \frac{N}{v} dU,$$

in which N represents the ordinate of the frequency curve at the centre of the strip, v the velocity, and dU the width of the strip. The

* *P.A.S.P.*, **41**, 344, 1929.

band contour can then be computed by summation, starting with the larger velocities and building up the band from its outer edge to its centre.

Fig. 2 illustrates the result of such a process applied to a distribution of velocities corresponding to an ordinary error curve where the mean velocity of ejection is 20 km./sec. and the effective spread in velocities is ± 14 k./m sec. approximately. Any other distribution of velocities may be treated in a similar way, and an investigation of the various possibilities leads to the following general conclusions.

1. If suitable assumptions be made concerning the distribution of velocities, almost any observed symmetrical contour can be accounted for.

2. If the frequency distribution of velocities occurs within a range appreciably smaller than half the width of the band (so that there are

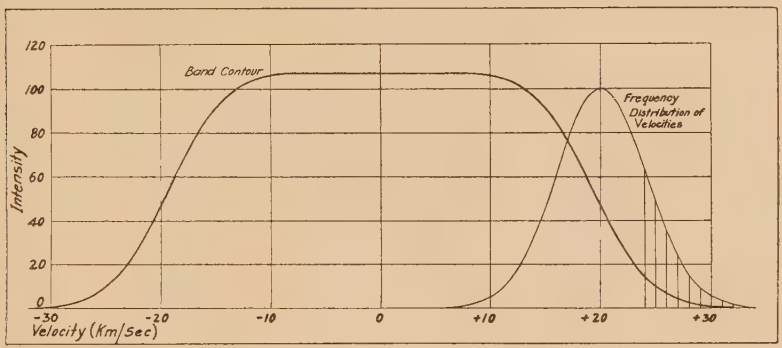

Fig. 2.

no velocities in the vicinity of zero), then the contour of the band will have a flat top (an example of this case is illustrated in fig. 2).

3. If the contour of the band be observed with sufficient accuracy, then it is possible to apply the converse process and determine the distribution of the velocities of the emitting atoms.

In a recent paper Miss Payne * has stated the physical consequences of equation (3) although she does not derive the formula. She has also called attention to conclusion (1) above.

It may be inferred from (2) that if the assumptions involved are justified, then one might reasonably expect to find flat-topped bands in the spectra of both novæ and Wolf-Rayet stars. The occurrence of other types of contours would also be expected, but would naturally be of less significance from the point of view of confirming the theory.

Evidence has already been presented by Miss Payne that flat-topped bands are present in the spectra of both Nova Pictoris and certain Wolf-Rayet stars. The present paper contains some additional evidence of the same sort derived from measures of the spectra of Nova Aquilæ, Nova Cygni, and three Wolf-Rayet stars.

* *H.B.*, **874**, 23, 1930.

Observational Data.

The observational data are represented by the seven graphs of figs. 3 and 4 and by Tables I. and II., which contain lists of the entries on the graphs. It should perhaps be mentioned that for the Wolf-Rayet stars having reasonably broad bands, the number of bands which are free from blends and which at the same time show reasonably strong contrast with the continuous spectrum are very few. The bands λ 4686, λ 4861, and λ 5695 appear to be the most promising, though λ 4686 probably has the band λ 4712 of neutral helium superposed on its less refrangible edge, while λ 4861 is probably a combination of $H\beta$ and λ 4859 of *He* II. As far as can be determined, λ 5695 is free from blends, though in the case of a band 100 A. wide it is naturally difficult to be certain. For the three bands of fig. 4 it seems reasonably certain that any blending effects which may be present are not serious enough to affect any conclusions which are drawn from the plotted contours. Descriptions of individual graphs are as follows :—

(a) *Novæ.*—Measurements of the band $H\beta$ in Nova Cygni and Nova Aquilæ are shown in the graphs of fig. 3. Unfortunately these plates were not standardized photometrically, and so the measures are given in terms of photographic density rather than absolute intensity. However, for the purpose of testing the particular point under discussion (the flatness of the top of the band), the density curves serve the purpose equally well.

The band of Nova Cygni is rather narrow, but its flat-topped character is well marked, and the evidence would appear even more convincing if a smaller scale of ordinates were employed. Three plates were measured, and each point on the graph is a normal from five observations.

The two graphs of the same band of Nova Aquilæ are plotted from measures of three plates each. Each point on the graph is a normal from six measures. All who are familiar with the details of the spectrum of this star will recall the complex structure shown by the bands at certain stages in its development. These finer details are obscured in these graphs and only the general outline of the band contour is represented. The two graphs leave little doubt of the general shape of the band as consisting of steeply sloping sides and a flat top. Indeed, at certain stages the bands of this star on a spectrogram approximate closely to rectangular strips of constant density, which is the form predicted by the simple theory of equation (3). The hump in the centre of the band observed on 1918 September 13–19 is real and not an effect of errors of measurement.

(b) *Wolf-Rayet Stars.*—The measures of Wolf-Rayet bands are represented by the graphs of fig. 4. With one exception they have been plotted from measures of standardized plates and so represent absolute intensities of the bands in terms of the neighbouring continuous spectrum. The standardization strips were impressed on the plate with the aid of a neutral tint wedge and an auxiliary spectrograph.

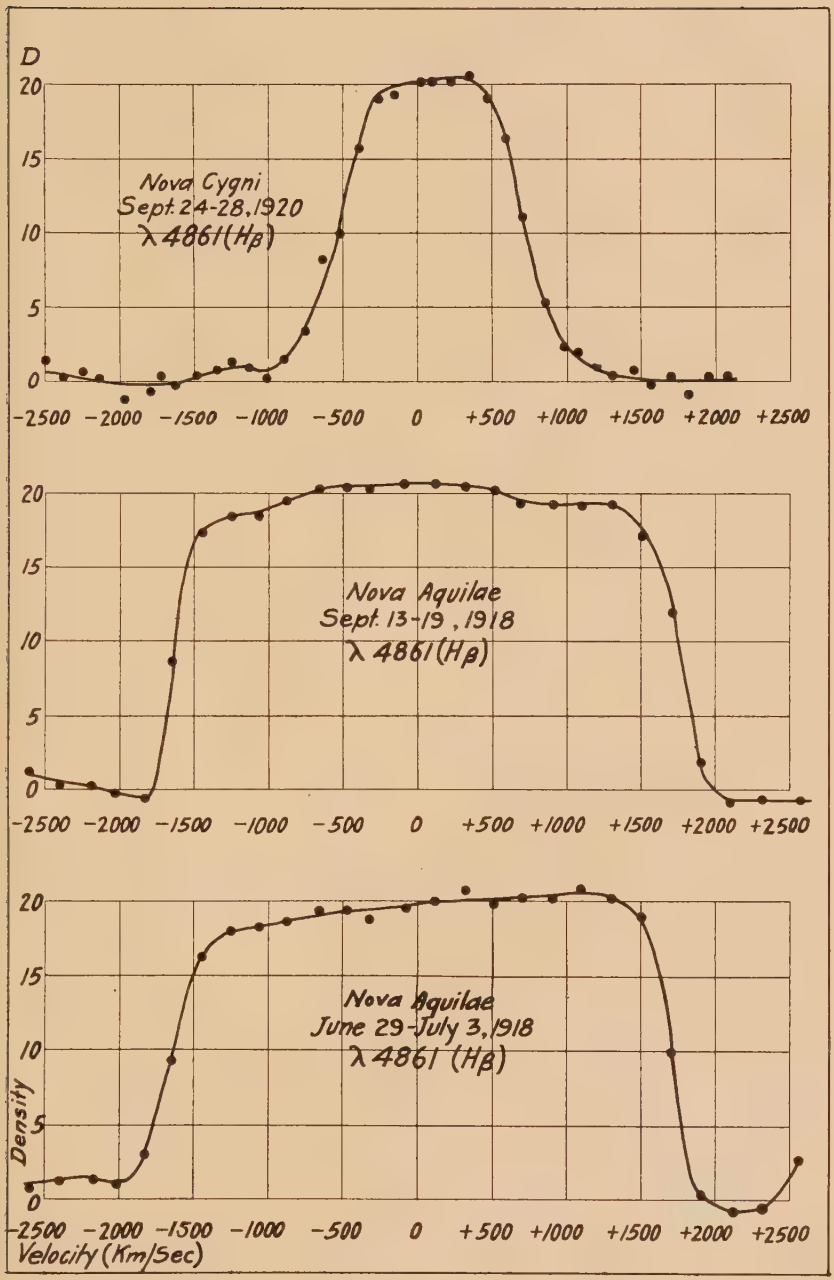
D
20
15
10
5
0
Nova Cygni
Sept. 24-28, 1920
λ 4861 (Hβ)
-2500 -2000 -1500 -1000 -500 0 +500 +1000 +1500 +2000 +2500
20
15
10
5
0
Nova Aquilae
Sept. 13-19, 1918
λ 4861 (Hβ)
-2500 -2000 -1500 -1000 -500 0 +500 +1000 +1500 +2000 +2500
20
15
10
5
0
Nova Aquilae
June 29-July 3, 1918
λ 4861 (Hβ)
Density
-2500 -2000 -1500 -1000 -500 0 +500 +1000 +1500 +2000 +2500
Velocity (Km/Sec)

FIG. 3.

The measures of density for both novæ and Wolf-Rayet stars were made with a Hartmann microphotometer.

The graph of the band λ 4686 in the spectrum of the star H.D. 191765 has been plotted from measures of five plates, and each normal is the mean of five observations. At one section of the curve the points show a considerable scatter, and this is believed to be due, not to errors of measurement, but to irregularities in the emulsion of two of the plates. The curve as a whole is well determined and there seems no reasonable doubt that the contour is practically flat on the top.

The next graph represents the band λ 4861 in the star H.D. 192163. Five plates were measured and each normal represents the mean of fifteen observations. The measures for this band were less consistent than those for the one just described, and this fact, combined with the narrowness of the top, makes the interpretation of the contour in terms of theory less certain. This band was, in fact, selected for measurement because it appeared to be a good example of absence of the flat-topped characteristic. It was somewhat of a surprise, therefore, when a final reduction of the observations gave some indications that a narrow section of the top of the band was flat.

The two lower graphs in the figure represent measures of the band λ 5695 in the spectrum of the star H.D. 193793. This band is approximately 100 A. in width, one of the widest appearing in any Wolf-Rayet star. Its origin is somewhat uncertain,* but it has been provisionally assigned to the atom of *C* III. Its great width would suggest the possibility of blending effects, but this idea is contradicted by the evenness of the contour and by the occurrence of other very wide bands in the spectrum of the same star. Also, in other stars where the same band appears in a narrower form there is no indication of other bands within 50 A. of the centre of λ 5695.

The upper graph of this star represents a series of measures made on two photometrically standardized plates. Each normal, on the average, represents the mean of four measures. Measurements of the two plates are in good agreement and clearly reveal the wide flat top of the band. In addition to the photometrically standardized plates, three other plates of this star were secured in the visible region before the equipment for standardizing plates was available. Since this band represents an important case, where the flat-topped characteristic appears to be established beyond possibility of doubt, it seemed worth while to combine density measures of these three plates with the first two. The density curve from measures of the five plates is shown by the lowest graph of fig. 4. Each normal of this graph is the mean of five measures. The second graph amply confirms the first in indicating a wide flat top to the band. It is of some interest to note that the second graph gives a clear indication of an absorption line on the violet edge of the band. The presence of another strong band to the red is indicated by the course of the graph to the right of the figure.

* A recent paper by Bowen (*Phys. Rev.*, **38**, 128, 1931) definitely identifies a line at 569 as due to *C* III.

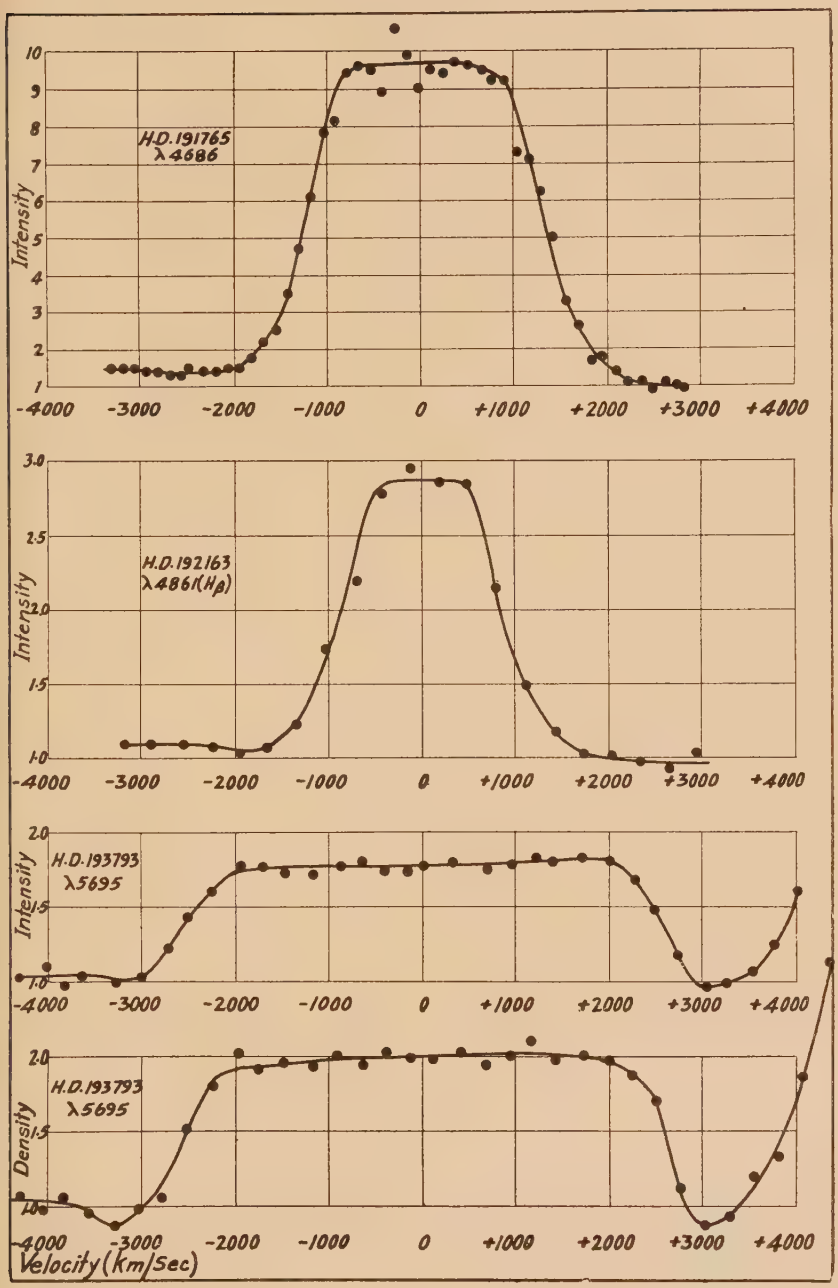
H.D.191765
λ4686
Intensity
10
9
8
7
6
5
4
3
2
1
-4000 -3000 -2000 -1000 0 +1000 +2000 +3000 +4000
H.D.192163
λ4861(Hβ)
Intensity
3.0
2.5
2.0
1.5
1.0
-4000 -3000 -2000 -1000 0 +1000 +2000 +3000 +4000
H.D.193793
λ5695
Intensity
2.0
1.5
1.0
-4000 -3000 -2000 -1000 0 +1000 +2000 +3000 +4000
H.D.193793
λ5695
Density
2.0
1.5
1.0
-4000 -3000 -2000 -1000 0 +1000 +2000 +3000 +4000
Velocity (Km/Sec)

Fig. 4.

TABLE I.

Entries on Graphs.

Nova Bands.

Nova Aquilæ. Hβ. June 29–July 3, 1918.		*Nova Aquilæ.* Hβ. Sept. 13–19, 1918.		*Nova Cygni.* Hβ. Sept. 24–28, 1920.			
Velocity.*	Density.	Velocity.	Density.	Velocity.	Density.	Velocity.	Density.
−3170	0·5	−3170	0·7	−2744	− 0·1	987	2·3
−2985	0·9	−2985	0·2	−2627	1·5	1085	2·0
−2787	0·9	−2787	0·7	−2504	1·4	1209	1·0
−2596	0·8	−2596	1·2	−2387	0·2	1314	0·4
−2405	1·2	−2405	0·3	−2257	0·6	1455	0·7
−2183	1·2	−2183	0·2	−2140	0·1	1573	− 0·3
−2023	0·9	−2023	− 0·2	−1973	− 1·2	1708	− 0·3
−1832	2·9	−1832	− 0·6	−1807	− 1·1	1813	− 0·9
−1640	8·7	−1640	8·6	−1721	0·4	1955	0·4
−1449	16·2	−1449	17·3	−1622	− 0·4	2084	0·4
−1252	17·9	−1252	18·4	−1492	0·4		
−1061	18·2	−1061	18·5	−1344	0·7		
− 870	18·5	− 870	19·5	−1246	1·5		
− 666	19·4	− 666	20·2	−1135	0·8		
− 475	19·3	− 475	20·3	−1011	0·1		
− 278	18·8	− 278	20·2	− 894	1·5		
− 80	19·8	− 80	20·7	− 759	3·3		
+ 117	19·9	+ 117	20·7	− 641	8·2		
+ 315	20·7	+ 315	20·4	− 524	9·9		
512	19·8	512	20·1	− 395	15·6		
709	20·1	709	18·7	− 259	18·9		
907	20·1	907	19·3	− 148	19·2		
1104	20·8	1104	19·3	− 19	20·1		
1307	20·2	1307	19·2	+ 99	20·1		
1505	18·9	1505	17·1	+ 228	20·1		
1702	9·7	1702	11·9	352	20·5		
1906	0·3	1906	1·8	463	18·7		
2109	− 0·8	2109	− 0·8	598	16·2		
2312	0·6	2312	− 0·6	709	11·0		
2559	2·6	2559	− 0·7	851	5·3		

* Velocities in these tables are given to the nearest kilometre per sec. for convenience in computation. The accuracy is actually much less and the error for a single observation may be as high as 20 km./sec.

TABLE II.

Entries on Graphs.

Wolf-Rayet Bands.

H.D. 191765. λ 4686.		H.D. 191765. λ 4686.		H.D. 191765. λ 4686.		H.D. 192163. λ 4861.	
Velocity.	Intensity.	Velocity.	Intensity.	Velocity.	Intensity.	Velocity.	Intensity.
−3314	1·5	− 659	9·6	2092	1·4	−3171	1·09
−3192	1·5	− 525	9·5	2227	1·1	−2893	1·08
−3065	1·5	− 397	8·9	2367	1·1	−2542	1·09
−2943	1·4	− 269	10·6	2495	0·9	−2246	1·07
−2815	1·4	− 141	9·9	2630	1·1	−1956	1·03
−2687	1·3	− 13	9·0	2764	1·0	−1653	1·07
−2566	1·3	+ 122	9·5	2828	0·9	−1345	1·22
−2438	1·5	+ 250	9·4			−1018	1·72
−2316	1·4	377	9·7			− 709	2·18
−2188	1·4	512	9·6			− 426	2·78
−2060	1·5	646	9·5			− 117	2·95
−1932	1·5	774	9·2			+ 204	2·86
−1811	1·8	902	9·2			+ 506	2·85
−1683	2·2	1036	7·3			802	2·14
−1555	2·5	1171	7·1			1123	1·49
−1427	3·5	1299	6·2			1431	1·17
−1299	4·7	1433	5·0			1734	1·02
−1171	6·1	1561	3·3			2042	1·01
−1043	7·8	1695	2·6			2332	0·97
− 915	8·1	1830	1·7			2653	0·92
− 787	9·4	1951	1·8			2937	1·02

H.D. 193793. λ 5695.		H.D. 193793. λ 5695.		H.D. 193793. λ 5695.		H.D. 193793. λ 5695.	
Velocity.	Intensity.	Velocity.	Intensity.	Velocity.	Density.	Velocity.	Density.
−4911	1·01	+ 326	1·79	−5722	−0·1	− 368	8·2
−4600	1·02	711	1·75	−5401	+0·2	− 116	7·9
−4317	1·02	969	1·78	−5106	0·1	+ 111	7·8
−4017	1·08	1221	1·83	−4833	0·8	+ 411	8·2
−3843	0·97	1400	1·80	−4596	0·1	705	7·5
−3643	1·04	1716	1·83	−4311	0·5	958	8·0
−3274	0·99	2011	1·80	−4043	−0·2	1184	8·8
−3011	1·02	2279	1·67	−3822	0·5	1437	7·8
−2732	1·23	2511	1·48	−3569	−0·3	1732	8·0
−2516	1·43	2758	1·17	−3301	−1·1	2006	7·7
−2258	1·59	3043	0·97	−3032	−0·2	2253	6·9
−1932	1·77	3264	0·99	−2779	0·4	2516	5·5
−1700	1·76	3537	1·06	−2516	4·1	2785	0·9
−1474	1·72	3780	1·06	−2243	6·4	3032	−1·1
−1174	1·71	4032	1·61	−1985	8·1	3280	−0·6
− 869	1·77			−1753	7·3	3559	1·6
− 637	1·79			−1479	7·6	3817	2·7
− 405	1·74			−1174	7·4	4085	6·9
− 137	1·74			− 895	8·0	4369	13·1
+ 11	1·77			− 637	7·5		

Discussion.

The main conclusions to be drawn from these observations appear to be reasonably obvious. The measurements of the bands of Nova Aquilæ and Nova Cygni, combined with the results of Miss Payne for Nova Pictoris, definitely establish the fact that flat-topped bands are a characteristic feature of the spectra of these two novæ at certain stages of their development. The agreement of the observed contours with the theoretical ideas developed in the first part of this paper provides additional confirmation, if such be necessary, of the ejection theory of novæ, which has held the field for a considerable number of years. The existence of flat-topped bands in Wolf-Rayet spectra also would appear to be in agreement with the theory that the emission bands in these stars are due to the continuous ejection of atoms from their surfaces. The fact that many, or even the majority, of Wolf-Rayet bands may not exhibit this characteristic is not a serious objection since the theory is able to account for any contour ordinarily observed.

If the theory be accepted, then it follows from the observed contours that the frequency distribution of velocities in each of the three Wolf-Rayet stars examined has a very considerable spread. The data of the present paper are not considered to be sufficiently extensive for a precise determination of the frequency curves of velocities for the ejected atoms. It is possible, however, from a study of the graphs to make a rough estimate of the maximum and minimum velocities for a given star. The maximum velocity, of course, corresponds to the extreme edge of the band. The minimum velocity corresponds to the edge of the flat portion of the top of the band. For the three stars of fig. 4 these values are approximately as follows :—

H.D. 191765	λ 4686	V. max. = 2400	km./sec.
		V. min. = 700	,,
H.D. 192163	λ 4861	V. max. = 1800	,,
		V. min. = 400	,,
H.D. 193793	λ 5695	V. max. = 3200	,,
		V. min. = 2000	,,

It will be observed that the minimum velocity for the star H.D. 192163 is 400 km./sec., a figure considerably less than the velocity of escape at the surface of the sun. There are almost undoubtedly other stars for which the minimum velocity is much less, perhaps even less than 100 km./sec. We have no knowledge of the value of gravity or the velocity of escape at the surface of a Wolf-Rayet star. It will presumably be of the order of a few hundred kilometres per second. Whatever its value, it seems entirely probable that there will be some stars from which considerable numbers of atoms will be shot off at velocities approximately equal to or less than the parabolic velocity of escape. Such atoms may come to rest at considerable distances from the star and remain in equilibrium under the action of whatever forces

are operative in the case of the atoms of planetary nebulæ. If conditions were sufficiently favourable, considerable accumulations of atoms might occur. Such a process offers a possible mechanism for the formation of planetary nebulæ. The existence of Wolf-Rayet stars as the nuclei of planetaries would appear to lend some probability to such a suggestion, though many such nuclei do not show the Wolf-Rayet characteristic.

A similar idea concerning the origin of planetaries was advanced by Menzel * in 1929. At that time the writer considered that the magnitude of the velocities involved was too great to render accumulation possible. The present work, by demonstrating that comparatively low velocities may occur even in a star having fairly wide bands, has made this difficulty less serious. It would appear, therefore, that this hypothesis, which suggests that the origin of planetary nebulæ may be due to the continuous ejection of atoms from the surface of a Wolf-Rayet star, while not without its difficulties, may still be worthy of serious consideration.

Dominion Astrophysical Observatory,
Victoria, B.C., Canada:
1931 *June* 4.

Selective Absorption. By R. v. d. R. Woolley, Ph.D.

In the quantum theory, as originally given by Einstein and Bohr, the absorption of light takes place as the result of the capture of a quantum by an atom, and in capturing the quantum one of the electrons in the atom jumps from a certain energy level A to a higher level B. If E_A, E_B are the energies of the two levels and ν the frequency of the quantum, then $E_B - E_A = h\nu$ where h is a universal constant. The atom has a certain discrete set of energy levels, and it is therefore only able to absorb a certain discrete set of frequencies, the frequencies characteristic of the spectrum. Absorption lines, however, are observed to have a finite width. This width can be made greater than the width due to the Doppler effect of the motions of the absorbing particles, and to the effect of pressure. An example of this is afforded by the B band of molecular oxygen, which appears sharp when observed with an air path of 10 metres at atmospheric pressure and room temperature. In the solar spectrum, where the absorption is due to a column of the earth's atmosphere, the lines appear much broader. It is clear that in this case the broadening cannot be attributed to anything except abundance of the absorbing atoms. The atom performs a certain amount of absorption at a frequency a small distance away from the central frequency of the line.

We are therefore led to the notion that the energy levels are not ideally sharp. This leads to the question, of great importance in the theory of the formation of absorption lines in solar and stellar

* Menzel, *loc. cit.*

atmospheres, as to whether an atom which has absorbed a quantum of frequency ν_1 which is not exactly equal to the frequency ν_0 of the line, is bound to re-emit a quantum of exactly the same frequency.

It is conceivable that this might be true in the case of principal lines, though it can only be approximately true in any other case. Suppose that the absorption in the line considered takes place as the result of an electron transition from a lower state A to a higher state B. In the case of a principal line, A is the normal or unexcited state of the atom, and it is possible that it is an ideally sharp state—that is to say, that all electrons in this state possess the same energy. The width of the absorption line must, therefore, be attributed to a lack of sharpness in the state B. Then when an atom absorbs a quantum of frequency $\nu_1 = \nu_0 + \Delta\nu$, the energy of the electron in state B is $E_B + h\Delta\nu$.

The subsequent re-emission takes place by an electron transition to the state E_A exactly, so that the frequency of the re-emitted quantum is ν_1, exactly equal to the frequency of the absorbed quantum. This is because the lower state being ideally sharp, the electron has no choice but to return to the precise energy level from which it started.

In any other line, however, the lower state cannot be sharp, since it is itself the upper state of a principal line, or of some other line. The electron in the upper state of such a line has a certain choice of energy levels within the lower state to which it can return, and the frequency of the quantum emitted will not, in general, be precisely equal to the frequency of the quantum absorbed.

These "energy levels within the lower state" are a continuous band of energy levels, concentrated, for the most part, within a range of frequency small compared with the differences in frequency of the discrete energy levels of the atomic states, in exactly the same way as absorption lines in an absorption spectrum, though possessed of a certain width (whose exact value depends on the manner in which the width is defined), are easily recognisable as discrete lines: the widths of the lines are, in general, small compared with the distances between one line and another. (Of course, this is not always so, *e.g.* in a band spectrum or at the head of a series in a line spectrum.)

Let us suppose that when an atom which is in thermodynamical equilibrium with radiation inside an enclosure has an electron in state A there is a certain probability ${}^{A}P(x)dx$ that the energy of the electron lies between $E_A + hx$ and $E_A + h(x + dx)$, where E_A is the mean energy of the state A. By "the state A" is meant the group of levels $A(x)$.

Similarly, ${}^{B}P(x)dx$ is the probability that an electron in state B has an energy between $E_B + hx$ and $E_B + h(x + dx)$.

Fig. 1 illustrates the levels $A(x_1)$ and $B(x_2)$ in the states A and B respectively. The thick lines are the central levels A(0) and B(0).

We have introduced these probabilities in order to clarify the notion of the levels within a state, but we shall work mostly with a different quantity, ${}^{A}W(x)dx$, the "weight" attached to the level $A(x)dx$, that is to say, to the level of an electron whose energy lies between $E_A + hx$ and $E_A + h(x + dx)$. These weights are not the weights as used in statistical mechanics, but are the statistical weights

multiplied by an absorption coefficient. The weight ${}^{A}W(x)$ is defined as follows: when an atom is in thermodynamical equilibrium with radiation inside an enclosure and when a quantum is absorbed by a transition from one of the lower levels $A(x)$ to one of the upper levels $B(x)$ there is a probability ${}^{A}W(x)dx$ that the energy of the lower level lies between $E_A + hx$ and $E_A + h(x + dx)$.

If the lower levels are equally efficient in absorption, then ${}^{A}W(x) = {}^{A}P(x)$.

Similarly, we define ${}^{B}W(x)$ as follows: when an electron in one of the levels $B(x)$ performs a transition to one of the levels $A(x)$, there is a probability ${}^{B}W(x)dx$ that the energy level of the electron in the upper state lies between $E_B + hx$ and $E_B + h(x+ dx)$.

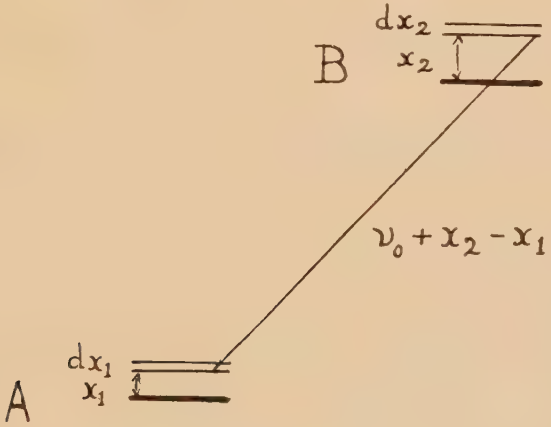

FIG. 1.

If the upper levels $B(x)$ all have the same excited life, then ${}^{B}W(x) = {}^{B}P(x)$.

It follows at once from the principle of detailed balancing that ${}^{A}W(x)dx$ is also the probability of a downward transition into the level $A(x)dx$ and that ${}^{B}W(x)dx$ is the probability of an upward transition into the level $B(x)dx$, but this is the probability of an upward transition from *all* the A levels, and the principle of detailed balancing does not inform us that it is also the probability of a transition into the level $B(x)dx$ from each individual level $A(x)dx$, that is to say, that when a quantum is absorbed by an atom whose electron is in the level $A(x_1)dx$, the probability $P(A(x_1) \rightarrow B(x_2))dx$ that the upper level of the electron after the transition will be $B(x_2)dx$ is equal to ${}^{B}W(x_2)dx$, *i.e.* is independent of x_1. If this is so, *i.e.* if

$$P(A(x_1) \rightarrow B(x_2))dx = {}^{B}W(x_2)dx. \quad . \quad . \quad (1)$$

then the absorption coefficient of the level $A(x)$ is shifted to the red if x is positive and to the violet if x is negative, the central frequency being $\nu_0 - x$.

Another transition scheme allowed by the principle of detailed balancing, namely, transitions between $A(x_1)dx$ and $B(x_2)dx$ occur only if $x_1 = x_2$, gives an infinitesimally narrow absorption line and is therefore ruled out.

The transition scheme obtained by giving each level a symmetrical absorption coefficient with a central frequency ν_0 ($= f(\nu - \nu_0)^2$, say) is ruled out by the principle of detailed balancing.

Let

$$N(A(x_1) \rightarrow B(x_2))dx$$

be the number of transitions per sec. from $A(x_1)dx$ to $B(x_2)dx$.

Then $N(A(x) \rightarrow B(x + h)) = {}^{A}P(x)f(h^2)$.

Also $N(A(x + 2h) \rightarrow B(x + h)) = {}^{A}P(x + 2h)f(h^2)$.

But $N(B(x + h) \rightarrow A(x)) = N(B(x + h) \rightarrow A(x + 2h))$

$(= {}^{B}P(x + h)f(h^2))$,

so that detailed balancing cannot be satisfied unless ${}^{A}P(x) = {}^{A}P(x + 2h)$, which cannot be true for all x unless the absorption line is infinitely broad.

The generalised form

$$ {}^{A}W(x_1)P(A(x_1) \rightarrow B(x_2))dx = {}^{A}W(x_1){}^{B}W(x_2)f(x_1 - x_2)^2dx \qquad (2)$$

satisfies the condition of detailed balancing. A physical meaning can be attached to the term $f(x_1 - x_2)^2$ if it decreases with increasing $|(x_1 - x_2)|$, namely, that transitions tend to occur more frequently between levels with nearly equal displacements (in the same sense) than between levels distributed at random (and suitably weighted for total numbers of transitions). We can, however, adduce some grounds for supposing that no such preference exists, and that $f(x_1 - x_2)^2 \equiv 1$.

Assuming for the moment that this is so, then the probability of a transition between $A(x_1)dx$ and $B(x_2)dx$ is proportional to

$$ {}^{A}W(x_1)dx_1{}^{B}W(x_2)dx_2 \qquad . \quad . \quad . \qquad (3)$$

We can now calculate the coefficient of absorption (except for a constant factor).

The frequency of the transition from $A(x_1)dx_1$ to $B(x_2)dx_2$ is

$$\nu_0 + x_2 - x_1.$$

Hence the absorption in the range ν_1 to $\nu_1 + d\nu$ is the result of transitions such that

$$x_2 - x_1 = \nu_1 - \nu_0,$$

with

$$dx_2 = dx_1 = \tfrac{1}{2}d\nu.$$

The probability that the quantum, absorbed in the line, is in the range ν_1 to $\nu_1 + d\nu$ is therefore proportional to

$$\int {}^{A}W(x_1)dx_1{}^{B}W(x_2)dx_2,$$

subject to

$$x_2 - x_1 = \nu_1 - \nu_0$$

and

$$dx_2 = dx_1 = \tfrac{1}{2}d\nu.$$

The probability is therefore proportional to

$$\tfrac{1}{2}d\nu \int_{-\infty}^{\infty} {}^{A}W(x + \nu_1 - \nu_0)\,{}^{B}W(x)dx.$$

If we put

$${}^{A}W(x) = {}^{B}W(x) = \frac{c/\pi}{c^2 + x^2} \quad . \quad . \quad . \quad (4)$$

this becomes

$$\int_{-\infty}^{\infty} {}^{A}W(x + \nu_1 - \nu_0)\,{}^{B}W(x)dx = \frac{c^2}{\pi^2}\int_{-\infty}^{\infty} \frac{dx}{(c^2 + x^2)(c^2 + (x + \nu_1 - \nu_0)^2)}$$

$$= \frac{2c}{\pi}\frac{1}{4c^2 + (\nu_1 - \nu_0)^2},$$

which is the Lorentz form of the scattering coefficient,

$$s_{\nu_1} = \frac{s_0}{1 + a^2(\nu_1 - \nu_0)^2},$$

if we put

$$4c^2 = 1/a^2,$$

so that

$${}^{A}W(x) = {}^{B}W(x) = \frac{2a/\pi}{1 + 4a^2x^2}.$$

We can now state the grounds for supposing that $f(x_1 - x_2)^2 \equiv 1$ in equation (2). ${}^{A}W(x)$ and ${}^{B}W(x)$ must decrease with increasing x at least as rapidly as is given by (4), in order to represent the physical idea of a set of levels distributed about a mean value. If $f(x_2 - x_1)^2$ (which must also decrease with increasing $| x_1 - x_2 |$ in order to represent the physical picture previously mentioned) appears as a factor (constant for constant $\nu_1 - \nu_0$ since $(x_2 - x_1)^2 = (\nu_1 - \nu_0)^2$) in (2), we will obtain a higher power of $\nu_1 - \nu_0$ in the denominator of the scattering coefficient. In order that our model of the atomic scattering process should give the Lorentz scattering coefficient, we are then compelled to adopt the form (3), *i.e.*

$$f(x_1 - x_2)^2 \equiv 1.$$

Other forms of $W(x)$ give different forms for the scattering coefficient. For example, if

$${}^{A}W(x) = e^{-\alpha x^2} \quad \text{and} \quad {}^{B}W(x) = e^{-\beta x^2},$$

then

$$s(\nu_1 - \nu_0) = s_0 \exp - \left\{\frac{\alpha\beta}{\alpha + \beta}(\nu_1 - \nu_0)^2\right\}.$$

Now suppose that the atom has absorbed a quantum of frequency ν_1.

We wish to calculate the probability $P_{12}d\nu$ that it will re-emit a quantum in the frequency range ν_2 to $\nu_2 + d\nu$.

Consider the absorption of the quantum of frequency ν_1. The probability that the corresponding transition occurred between the range $A(x)dx$ and $B(x + \nu_1 - \nu_0)dx$ is proportional to

$$^{A}W(x)\,^{B}W(x + \nu_1 - \nu_0)dx.$$

The probability that the re-emission will be a transition to $A(x_2)dx_2$ is proportional to

$$^{A}W(x_2)\,^{A}W(x)\,^{B}W(x + \nu_1 - \nu_0)dx_2dx.$$

The re-emission will be in the required range if

$$(x + \nu_1 - \nu_0) - x_2 = \nu_2 - \nu_0 \qquad \text{or} \qquad x_2 = x_1 + \nu_1 - \nu_2,$$

with

$$dx_2 = d\nu.$$

Hence

$$P_{12} = \int_{-\infty}^{\infty} {}^{A}W(x) \,.\, {}^{A}W(x + \nu_1 - \nu_2)\,{}^{B}W(x + \nu_1 - \nu_0)dx\,. \qquad (5)$$

With

$$^{A}W(x) = {}^{B}W(x) = \frac{c/\pi}{c^2 + x^2}$$

this leads to

$$P_{12} = \frac{\frac{2c}{\pi}\{(\nu_1 - \nu_2)^2 + (\nu_1 - \nu_0)(\nu_2 - \nu_0) + 12c^2\}}{\{4c^2 + (\nu_1 - \nu_0)^2\}\{4c^2 + (\nu_2 - \nu_0)^2\}\{4c^2 + (\nu_2 - \nu_1)^2\}} \qquad (6)$$

P_{12} has maxima at $\nu_1 - \nu_2 = 0$ and $\nu_0 - \nu_2 = 0$ (ν_1 being fixed). When a quantum of frequency ν_1 has been absorbed, the subsequent re-emission is most likely to occur in a small range of frequencies centred about ν_1 or in a similar range centred about the central frequency ν_0.

There is a simple explanation of this result. If a quantum of frequency ν_1 to $\nu_1 + d\nu$ has been absorbed, the probability that the transition took place from the lower level $A(x)dx$ is proportional to

$$^{A}W(x)\,^{B}W(x + \nu_1 - \nu_0)dx,$$

i.e. to

$$\frac{1}{c^2 + x^2}\,\frac{1}{c^2 + (x + \nu_1 - \nu_0)^2}dx,$$

which has maxima at $x = 0$ and $x = -(\nu_1 - \nu_0)$.

The most probable transitions for the absorption of the ν_1 frequency are therefore those which start from the centre of the A levels, and those which go to the centre of the B levels. The transitions which start from the centre of the A levels go to $B(\nu_1 - \nu_0)$, from which the most probable re-emission is a return to A(0), emitting the original frequency ν_0. Those which start from $A(-\nu_1 - \nu_0)$, on the other hand, go to B(0). The most probable re-emission is again the return to A(0), which gives in this case the central frequency ν_0. Fig. 2 shows the two

most probable transitions giving the absorption of the frequency ν_1, with consequent re-emission of ν_0 and ν_1 respectively.

The two branches $\nu_2 = \nu_1$ and $\nu_2 = \nu_0$ are equally probable, *i.e.* $P_{11} = P_{10}$. The ranges of re-emission are small compared with the width of absorption lines usually dealt with in Astrophysics. For example, consider the case where $\nu_1 - \nu_0$ is 10^{10}. (This corresponds to a distance in wave-length of about ·05 A.) $2c$ is of order 10^7, so that P_{12} becomes approximately

$$\frac{\text{const.}}{4c^2 + \Delta\nu^2}$$

at a distance $\Delta\nu$ from the centre of either component. It is reduced to $\frac{1}{100}$ of its central value in the frequency range $\Delta\nu = 10^8$.

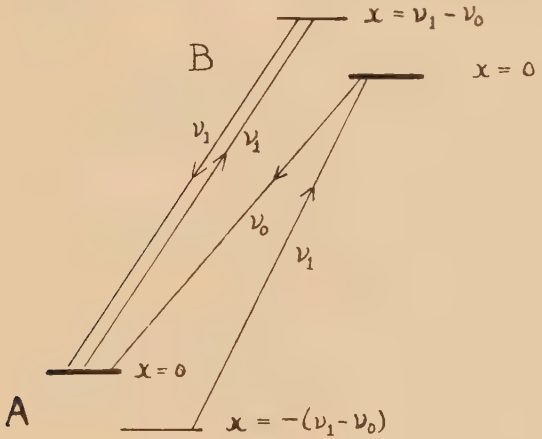

FIG. 2.

We have deduced our expression for P_{12} by an investigation of thermodynamical equilibrium inside an enclosure. Some modification may occur when the conditions no longer conform to this state.

The weight ${}^{A}W(x)$ is defined as the number of transitions from the level $A(x)$ in thermodynamical equilibrium with radiation in an enclosure (black-body radiation). We have used it, however, both in determining the scattering coefficient and in calculating P_{12} as the product of the quantity ${}^{A}P(x)$ and an appropriate scattering coefficient for the level, so that under any conditions ${}^{A}W(x)$ is a constant multiplied by ${}^{A}P(x)$ when considering any particular frequency ν. ${}^{A}P(x)$ will, in general, depend on the radiation field if A is the upper state of a line. In particular, it will not have its black-body value at the outer layers of a stellar atmosphere. It is a consequence of this result that the atomic scattering coefficient s_ν (as distinct from the scattering coefficient of a *level*) is also dependent on the radiation field. P_{12} is of course also dependent on the field, except in the case of principal lines, for which both s_ν and P_{12} are independent of the radiation field.

In a laboratory experiment, such as that suggested in the next paragraphs, P(x) for the lower state can be made to have its normal value if suitable precautions are observed as to the manner in which the atom is excited into this state. We can then apply our formula for P_{12} although the conditions differ widely from thermodynamical equilibrium.

The foregoing theoretical discussion, based on a physical model of the process of line absorption and the principle of detailed balancing, leads us to expect that if a vapour of some substance is illuminated by monochromatic light of a frequency slightly different from the frequency of a spectrum line of the substance, the spectrum of the scattered radiation will be a doublet, the two frequencies being the frequency of the incident light and the line frequency. Principal lines may form an exception to this rule.

An experimental test of this conclusion is possible. A simple qualitative examination of the nature of scattered radiation when the frequency of the incident light does not include a line frequency of the vapour could be made by illuminating a vapour with a source of light showing one of its spectrum lines strongly reversed, by a separate cell of the vapour if necessary. The scattered light should not exhibit the reversal, according to the results of the present paper. There are grave experimental difficulties, however. Resonance radiation is feeble even under the most favourable conditions, and very long exposures might be required with a reversed source of excitation. It is of course difficult to detect resonance radiation in the laboratory with any lines except principal lines, even without a reversed source. Nevertheless, it is hoped to carry out some such experiments at the Solar Physics Observatory, Cambridge, in the near future.

It would be unwise to predict the effect of the phenomenon on the contours of absorption lines, without a detailed examination. The writer hopes to make such a detailed examination on another occasion.

The writer wishes to acknowledge helpful criticism by Mr. G. G. Wiles.

1931, *August* 6.

The Effect of Opacity in the Point-source Stellar Model.

By J. C. P. Miller, B.A.

(*Communicated by Sir A. S. Eddington.*)

In his recent theory of stellar structure, Milne uses models in which matter can liberate energy only at very high densities and temperatures; 10^9 degrees or more may perhaps be considered the order of the latter.

For the outer parts of such a star, *i.e.* where $T < 10^9$, a point-source model is applicable, and Eddington has already considered such a model.*

* *Internal Constitution of the Stars*, § 91, p. 124; Table 11, p. 126.

In this particular case, however, the temperature has only reached about $5{\cdot}5 \times 10^6$ degrees when the density begins to diminish as we proceed inwards, and before this point the solution ceases to be physically possible. For this solution to apply, generation of energy must set in before the density reaches a maximum, and so at a temperature too low for this model to be applicable to Milne's work. The variation of density ρ with distance r from centre in Eddington's solution is represented by the curve I in the diagram.

Eddington's model has the opacity law

$$k = k_1 \rho / T^{\frac{7}{2}},$$

k_1 being a constant. What we wish to find out is whether it is possible to find a model with the same radius, mass, and luminosity, but with a sufficient increase in k_1, such that a temperature of the order 10^9 is reached before the solution ceases to be physically possible; and if so, how great an increase in k_1 is required. In this paper we obtain such a model, and so an upper limit to the required value of k_1.

Strömgren * has shown that for $k_1 > k_1^0$ (the value in Eddington's model) we must have $\rho \to \infty$ and $T \to \infty$ as $r \to 0$, so that such models will certainly have a region with $T > 10^9$. We do not expect, however, that the maximum of ρ will immediately disappear. In fact, for k_1

* *Zeitschrift für Astrophysik*, **2**, 345, 1931.

very little greater than k_1^0, we expect ρ and r to be related somewhat as represented by II in the diagram. This model (II) will still not be physically possible throughout, having a region with ρ decreasing inwards. We must increase k_1 until this is no longer the case, as in III, where ρ has a horizontal inflection. We must, in fact, increase k_1 still further until the stricter stability condition

$$\gamma' = d(\log p_G)/d(\log \rho) \not> \tfrac{5}{3}$$

is satisfied everywhere. If k_1^1 is the value of k_1 for the model in which γ' has a maximum of exactly $\frac{5}{3}$, then models with $k_1 > k_1^1$ may be used right up to the required temperature (10^9) before generation of energy sets in.

It might happen that a value rather less than k_1^1 would suffice if the temperature reached 10^9 degrees whilst γ' was less than $\frac{5}{3}$ but was still increasing (with decreasing r) to a maximum greater than $\frac{5}{3}$. But this is very unlikely; though we may perhaps expect the temperature at which generation of energy must set in to increase a little with k_1, it will be, at any rate, less than that of the shell where γ' has its maximum (as temperature increases inwards), and in the model given in this paper the latter temperature is about $1{\cdot}7 \times 10^7$ degrees.

In our model (with density curve represented by IV) γ' reaches a maximum $1{\cdot}328$ at about $r = 7{\cdot}8 \times 10^{10}$ cm., and this maximum is considerably less than $\frac{5}{3}$ (the slightly greater value $1{\cdot}333$ at $r = 8{\cdot}1 \times 10^{11}$ is probably due to a slightly incorrect boundary condition). So in this case k_1 is certainly greater than k_1^1, probably considerably so, as the inflections which occur in II and III do not exist in IV. The model, apart from shells which may be considered as surface layers, was chosen to have approximately the same radius, luminosity, and mass as Eddington's model, but with $k_1 = 10k_1^0$.

We conclude that for a star of the mass here considered (about $5 \times \odot$) the opacity required by Milne's theory, whilst greater than that required by Eddington's theory, is not very much greater, since our tenfold increase covers the conditions with a wide margin to spare.

We also notice that besides models which, with Milne's ideas, may be called collapsed ($k_1 < k_1^0$), and others which may be called centrally condensed ($k_1 \geqslant k_1^1$), there are others ($k_1^0 \leqslant k_1 < k_1^1$) in which generation of energy must set in when the temperature is very much lower than the temperatures required by Milne's theory.

Equations and Method.—The variables and constants here are those used by Eddington.

The equations are

$$dP = -g\rho dr \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$dp_R = kLdP/4\pi cGM_r \quad . \quad . \quad . \quad . \quad (2)$$

$$= -k_1 L\rho^2 dr/4\pi cr^2 T^{\frac{7}{2}} \quad . \quad . \quad . \quad (3)$$

$$T^4 = \frac{3}{a}p_R \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$\rho = \frac{\mu}{\Re}p_G/T = \frac{\mu}{\Re}(P - p_R)/T \quad . \quad . \quad (5)$$

$$dM_r = 4\pi\rho r^2 dr \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The initial data used are, at $r = 8{\cdot}1 \times 10^{11}$ cm.,

$$M_r = 9{\cdot}699 \times 10^{33}$$
$$P = {\cdot}02455 \times 10^{12}$$

and the condition that $\frac{d}{dr}(p_G/p_R) = 0$ at this shell. Strictly we should use $\frac{d}{dr}(p_G/p_R) = 0$ at the boundary, but we are here neglecting the surface layers, and the difference is slight, as Eddington has shown. We use this condition directly, instead of using first $M_0 = 0$, which is no longer necessary, and satisfying $\frac{d}{dr}(p_G/p_R) = 0$ by trial.

We put

$$z = p_R/P \quad \text{and} \quad \sigma = \frac{kL}{4\pi cGM_r}\frac{p_R}{p_G} = \frac{\mu a}{3\Re}\frac{k_1 L T^{-\frac{1}{2}}}{4\pi cGM_r}$$

and substitute in (2),* giving

$$\frac{zdz}{z^2 + \sigma z - \sigma} = -\frac{dP}{P}.$$

σ is no longer constant as in Eddington's case,* but this does not affect the argument.

The condition $dz = 0$ thus becomes

$$z^2 + \sigma z - \sigma = 0 \quad . \quad . \quad . \quad . \quad (7)$$

as $dP \neq 0$.

This gives z and so the starting values of p_R and p_G when we know T.

From (6)

$$z = -\tfrac{1}{2}(\sigma - \sqrt{\sigma^2 + 4\sigma}), \quad \text{the positive root of (7),}$$
$$= \frac{ak_1 L}{2T^{\frac{1}{2}}}\left(\sqrt{1 + \frac{4}{ak_1 L}T^{\frac{1}{2}}} - 1\right)$$

where $a = \mu a/12\pi c\Re GM_r$.

Also $p_R = zP$, so that $T^4 = \frac{3}{a}p_R = \frac{3}{a}Pz$, whence we derive an equation for T:—

$$T^{\frac{9}{2}} = \frac{3}{a}P\frac{ak_1 L}{2}\left(\sqrt{1 + \frac{4}{ak_1 L}T^{\frac{1}{2}}} - 1\right) \quad . \quad . \quad (8)$$

This we solve by successive approximations.

* *Cf.* Eddington, *M.N.*, **90**, 279, 1930.

For Eddington's model we have, at $r = 8{\cdot}1 \times 10^{11}$,

$$M_r = 9{\cdot}699 \times 10^{33}, \qquad P = {\cdot}02455 \times 10^{12}, \qquad k_1 L = 4{\cdot}560 \times 10^{62},$$

whence $T = 1{\cdot}285 \times 10^6$; Eddington has $1{\cdot}28 \times 10^6$.

It will be seen from the following figures that the stationary point of z in Eddington's model is at about $r = 7{\cdot}7 \times 10^{11}$ and not at $r = 8{\cdot}1 \times 10^{11}$:—

$r/10^{11}$	8·1	7·7	7·3	6·9	6·5	6·1
z	·275	·279	·275	·271	·268	·266

From (7) and (8) we find $z = {\cdot}2837$.

In the present case we take, at $r = 8{\cdot}1 \times 10^{11}$,

$$M_r = 9{\cdot}699 \times 10^{33}, \qquad P = {\cdot}02455 \times 10^{12}, \qquad k_1 L = 4{\cdot}560 \times 10^{63}.$$

Whence $T = 1{\cdot}5640 \times 10^6$.

r. 10^{11}	M_r. 10^{33}	ρ. 10^{-2}	T. 10^6	p_G. 10^{12}	p_R. 10^{12}	γ'. Unit.
·16	3·32615	1328·3	57·961	28895	28750	1·3175
·18	3·40931	994·84	52·873	19741	19907	1·3183
·2	3·48845	769·86	48·724	14078	14357	1·3191
·22	3·56430	611·52	45·269	10389·3	10697·1	1·3199
·26	3·70808	409·79	39·817	6123·7	6402·5	1·3213
·3	3·84352	291·61	35·687	3905·6	4131·4	1·3225
·34	3·97238	216·85	32·430	2639·3	2817·5	1·3236
·38	4·09584	166·75	29·784	1864·0	2004·5	1·3245
·42	4·21468	131·66	27·583	1362·9	1474·4	1·3253
·46	4·32947	106·17	25·717	1024·7	1114·2	1·3259
·5	4·44065	87·128	24·111	788·41	860·87	1·3264
·6	4·70475	56·377	20·913	442·47	487·20	1·3274
·7	4·95162	38·823	18·507	269·65	298·82	1·3278
·8	5·18344	27·955	16·618	174·35	194·26	1·3278
·9	5·40171	20·819	15·088	117·88	131·99	1·3276
1·0	5·60759	15·916	13·818	82·538	92·857	1·3272
1·1	5·80202	12·428	12·744	59·442	67·189	1·3267
1·2	5·98580	9·8751	11·822	43·816	49·762	1·3261
1·3	6·15964	7·9621	11·0215	32·934	37·587	1·3254
1·5	6·48004	5·3649	9·6953	19·521	22·507	1·3240
1·7	6·76780	3·7557	8·6396	12·178	14·192	1·3225
1·9	7·02685	2·7098	7·7781	7·9104	9·3234	1·3211
2·1	7·26059	2·0040	7·0613	5·3107	6·3330	1·3197
2·3	7·47199	1·5128	6·4554	3·6651	4·4234	1·3184
2·5	7·66366	1·1623	5·9366	2·5896	3·1638	1·3172
2·7	7·83784	·90667	5·4875	1·8672	2·3098	1·3161
2·9	7·99651	·71677	5·0952	1·3706	1·7168	1·3151

r. 10^{11}	M_r. 10^{33}	ρ. 10^{-2}	T. 10^6	p_G. 10^{12}	p_R. 10^{12}	γ'. Unit.
3·1	8·14138	·57339	4·7497	1·0221	1·2964	1·3142
3·3	8·27395	·46358	4·4433	·77305	·99284	1·3133
3·7	8·50731	·31155	3·9244	·45886	·60417	1·3117
4·1	8·70526	·21613	3·5025	·28410	·38336	1·3103
4·5	8·87452	·15401	3·1536	·18227	·25193	1·3091
4·9	9·02031	·11229	2·8606	·12055	·17058	1·3080
5·3	9·14677	·083525	2·6117	·081869	·118516	1·3072
5·7	9·25715	·063229	2·3979	·056902	·084222	1·3064
6·1	9·35407	·048616	2·2126	·040370	·061051	1·3058
6·5	9·43964	·037906	2·0506	·029172	·045040	1·3054
6·9	9·51559	·029933	1·9079	·021433	·033750	1·3058
7·3	9·58335	·023921	1·7811	·015990	·025636	1·3078
7·7	9·64412	·019344	1·6675	·012106	·019693	1·3144
8·1	9·699	·015856	1·5640	·009307	·015243	1·3333

We can now derive our starting values in full. The full solution is given in the accompanying table, which may be compared with Eddington's table (*loc. cit.*).

The method of integration used was to obtain a set of series giving the first few values of the various quantities, and then to proceed with the help of the formula

$$\Delta f = \frac{w}{2}(f_0' + f_1' - \tfrac{1}{2}\overline{\Delta_0^2 + \Delta_1^2 f'} + \tfrac{11}{720}\overline{\Delta_0^4 + \Delta_1^4 f'} - \ldots)$$

guessing f_1', $\Delta_1^2 f'$, etc., in advance, and choosing the interval sufficiently small for errors in the guesses not to affect Δf. The interval used (with 10^{11} as unit) was ·1 from $r = 8{\cdot}1$ to $r = 2{\cdot}1$, ·05 to $r = {\cdot}8$, ·02 to $r = {\cdot}32$, and ·01 to $r = {\cdot}16$.

The integration has been carried well beyond the maximum of γ'. For comparison we give the following table of γ' in Eddington's model:—

r.	γ'.	r.	γ'.	r.	γ'.
·5	+ ·937	2·9	+2·46	5·7	+1·340
1·3	+ ·800	3·3	+1·740	6·5	+1·318
2·1	+ ·272	3·7	+1·542	7·3	+1·318
2·5	−4·3	4·1	+1·450	8·1	+1·358
		4·9	+1·370		

This shows that generation of energy must begin in Eddington's model, at a shell distant more than about $3{\cdot}5 \times 10^{11}$ cm. from the centre.

I am under great obligation to Sir Arthur Eddington for his suggestion of the problem and help in tackling it.

Space Distribution of the Boss Stars. By R. D. H. Jones, B.A.

1. Some years ago Sir Frank Dyson * published two papers discussing the space distribution of the stars from the data of Carrington's Circumpolar Catalogue. In the first of these papers the observed distribution of proper motion components along a great circle, approximately at right angles to the solar motion and to the star streams, was compared with the theoretical implications of the proposed law of stellar density

$$D(r) = \frac{A}{r^{\lambda}} \cdot e^{-\kappa^2 r^2} \quad . \quad . \quad . \quad . \quad (1)$$

It was found that this law accounted in a satisfactory manner for the observed proper motion statistics, the value of λ being nearly equal to unity.

In the second paper the distribution of proper motion components projected along a great circle in the direction of the solar motion and that of star streaming was compared with the theoretical distribution given by the law

$$D(r) = \frac{A}{r} \cdot e^{-\kappa^2 r^2} \quad . \quad . \quad . \quad . \quad (2)$$

Dyson's investigation therefore dealt with a particular region of the sky, in which the proper motions could be resolved into two directions at right angles, such that one gave a random distribution of velocities, while the other gave a random distribution superimposed on the common drift motion.

The theoretical implications of the law (2) applicable to any region of the sky and to the distribution of proper motions in any position angle have been discussed by Dr. W. M. Smart, using the two-stream theory of stellar movements † and that of Schwarzschild.‡ On the basis of the former theory three methods were developed of comparing the theoretical and observed proper motion statistics :

(*a*) The distribution of magnitudes of proper motions in a selected position angle was considered.

(*b*) The form of the drift curves in any region was obtained, attention being restricted to those stars whose proper motions exceed 2″, 3″, etc. per century.

(*c*) The distribution of magnitudes of proper motions in any region was considered irrespective of position angle.

The present paper contains an account of an attempt to use methods (*b*) and (*c*) for testing Dyson's law by means of the data of Boss's Preliminary General Catalogue. Method (*a*) could not be used, as the observed statistics were not sufficiently extensive.

The theoretical distributions with which we are concerned in (*b*) and (*c*) depend not only on the form adopted for the law of stellar density, but also upon the constants of the star streams in the regions

* *M.N.*, **73**, 334, 402, 1913. † *M.N.*, **88**, 567, 1928.
‡ *M.N.*, **89**, 105, 1928.

under consideration. I have determined these constants for the same regions of the sky as those which we shall be discussing in this paper, the data being drawn from Boss's Catalogue.

The celestial sphere has been divided up into four zones parallel to the galactic equator, and each zone has been subdivided into two or more regions by meridians of galactic longitude. A table giving the extent of each region will be found in a previous paper.*

2. Consider a region of the sky in which there are N stars forming a drift † of which the velocity is hV.

Let $N(\mu, \theta) \, . \, d\theta$ denote the number of stars with proper motions greater than or equal to μ in directions between θ and $\theta + d\theta$, where θ is measured from the direction of hV.

It is convenient to express the law of stellar density in the form

$$D(r) = \frac{A}{r} \, . \, e^{-h^2k^2r^2}$$

which is essentially the same as Dyson's form.

Put

$$\eta = \frac{\mu}{(\mu^2 + k^2)^{\frac{1}{2}}}$$

and

$$\tau = h\text{V} \cos\theta.$$

Let $f(y)$ be the function introduced by Sir Arthur Eddington, defined by

$$f(y) = \frac{2}{\sqrt{\pi}} \, . \left\{ y \, . \, e^{y^2} \int_{-y}^{\infty} e^{-y^2} dy + \tfrac{1}{2} \right\}$$

Then we may derive the relation

$$N(\mu\theta) = \frac{N}{2\sqrt{\pi}} \, . \, e^{-h^2\text{V}^2} \, . \, \{ f(\tau) - \eta^2 f(\eta\tau) \} \quad . \quad . \quad (3)$$

It was observed by Dr. Smart that the curves given by the above form for $N(\mu, \theta)$ for any combination of the constants involved showed so close a resemblance to the ordinary frequency curves given by

$$\rho = \frac{N}{2\sqrt{\pi}} e^{-h^2\text{V}'^2} \, . f(\tau') \quad . \quad . \quad . \quad (4)$$

where

$$\tau' = h\text{V}' \cos\theta$$

that in all practical applications it was sufficiently accurate to use the simpler form (4)—with another value hV′ for the drift velocity—instead of the more complicated form (3). This quantity hV′ we shall refer to as the pseudo-drift velocity ; it is a function of hV and η—that is to say, of hV, μ, and h.

* *M.N.*, **91**, 563, 1931.

† For convenience I have included the short account contained in this section of the equations obtained by Dr. Smart, representing his distributions (*M.N.*, **88**, 569 *et seq.*, 1928).

If $R(\mu)$ denotes the number of stars in a region with proper motions equal to or greater than μ, then it may be shown * that $R(\mu)$ is related to the total number, N, of stars in the region by the equation

$$R(\mu) = N\{1 - \eta^2 \cdot e^{-h^2V^2(1-\eta^2)}\} \quad . \quad . \quad (5)$$

In applying formulæ (3) and (4) we are usually concerned with two drifts in directions θ_1, θ_2, say. Suppose the numbers of stars in the drifts are N_1, N_2 respectively and that the drift velocities are hV_1, hV_2, while the pseudo-drift velocities are hV_1', hV_2', then

$$N(\mu, \theta) = \frac{N_1}{2\sqrt{\pi}} \cdot e^{-h^2V_1'^2} \cdot f(\tau_1') + \frac{N_2}{2\sqrt{\pi}} \cdot e^{-h^2V_2'^2} \cdot f(\tau_2') \quad . \quad (6)$$

where

$$\tau_1' = hV_1' \cdot \cos(\theta - \theta_1)$$

and

$$\tau_2' = hV_2' \cdot \cos(\theta - \theta_2).$$

Also we have

$$R(\mu) = N_1\{1 - \eta^2 \cdot e^{-h^2V_1^2(1-\eta^2)}\} + N_2\{1 - \eta^2 \cdot e^{-h^2V_2^2(1-\eta^2)}\} \quad (7)$$

Equation (6) provides the means of testing Dyson's law by method (*b*), while (7) is the fundamental relation for method (*c*).

3. *Method* (*b*).—The effect on the form of the drift curves of restricting our attention to those stars in a region whose proper motions exceed some fixed quantity is to make the maxima more pronounced in comparison with the other portions of the curve. This is illustrated by figs. 1 and 2, which show the curves for the region N. In fig. 1 we have the frequency curves for all stars in the region, no restriction being placed on the magnitude of their proper motions, while fig. 2 refers to those stars for which $\mu \geqslant 4''$. The curves are analysed in the usual way; we find the values of the elements N_1, θ_1, hV_1', N_2, θ_2, hV_2' of the star streams which give the closest fit between the observed statistics and the theoretical form of the curve given by (6). The observed statistics are represented by a jagged line, while the theoretical relation giving the best fit is shown by a smooth curve. We find for the region N the following constants.

For unrestricted proper motions:

$\theta_1 = 325°$	$hV_1 = 1{\cdot}2$	$N_1 = 248$
$\theta_2 = 180°$	$hV_2 = 0{\cdot}65$	$N_2 = 197.$

For proper motions $\geqslant 4''$ per century:

$\theta_1 = 330°$	$hV_1' = 1{\cdot}55$	$N_1 = 148$
$\theta_2 = 170°$	$hV_2' = 0{\cdot}8$	$N_2 = 65.$

The complete results of this analysis for the seventeen regions of the sky are contained in Table I. below, for both restricted and unrestricted proper motions. A blank space in the table indicates that the element concerned cannot be found with any degree of certainty.

* W. M. Smart, *M.N.*, **88**, 580, 1928.

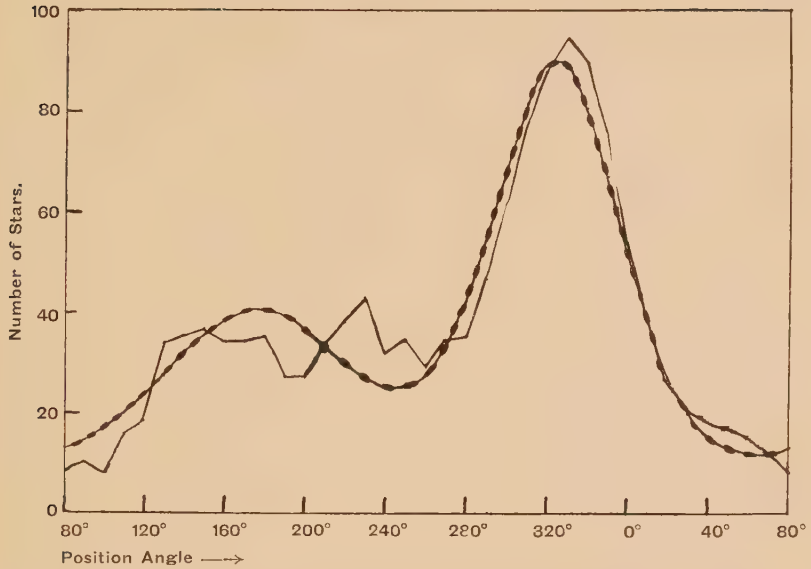

FIG. 1.—Region N ; Drift Curve for Unrestricted Proper Motions (445 stars).

Drift I. $\theta_1=325°$ $hV_1=1{\cdot}2$ $N_1=248$.
Drift II. $\theta_2=180°$ $hV_2=0{\cdot}65$ $N_2=197$.

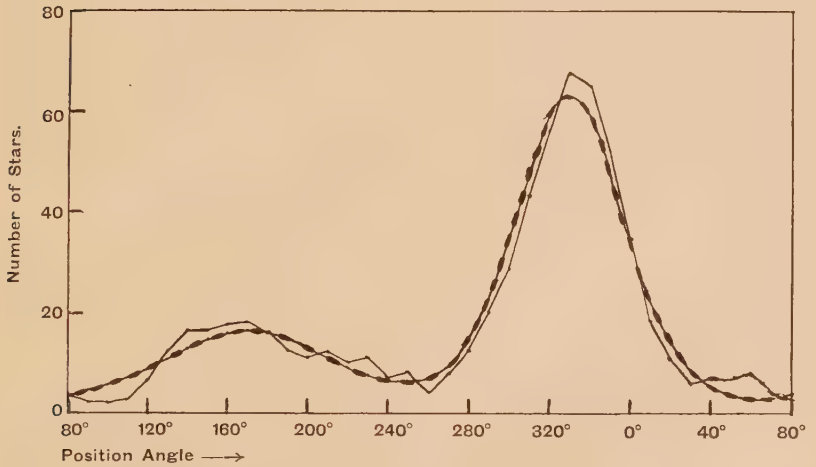

FIG. 2.—Region N ; Drift Curve for Stars having $\mu \geqslant 4''$ (213 stars).

Drift I. $\theta_1=330°$ $hV'_1=1{\cdot}55$ $N_1=148$.
Drift II. $\theta_2=170°$ $hV'_2=0{\cdot}8$ $N_2=65$.

TABLE I.

Results of Analysis of the Drift Curves.

	Region.	Drift I. $h\mathrm{V}_1$	Drift I. $h\mathrm{V}_1'$	Drift II. $h\mathrm{V}_2$	Drift II. $h\mathrm{V}_2'$
Zone I.	H	1·6	2·0	0·5	0·6
	I	1·3	1·7	0·4	0·6
	J	0·7	0·8	1·0	1·7
	K	1·3	1·7	0·8	1·2
Zone II.	E	1·5	—	—	—
	G	1·6	1·9	0·8	1·3
	L	1·7	1·9	—	0·6
	M	0·5	0·5	1·0	1·2
	N	1·2	1·55	0·65	0·8
Zone III.	B	1·2	1·4	0·5	0·35
	C	1·2	1·2	0·6	0·7
	D	1·4	1·6	0·8	0·9
	P	1·4	1·5	0·7	0·95
	Q	0·9	1·0	0·8	1·0
	R	1·1	1·3	0·75	0·7
Zone IV.	A	1·4	1·5	0·8	0·9
	S	1·2	1·3	0·75	0·9

It will be observed that, with the exceptions of Drift II in regions B and R, the drift velocities for the unrestricted proper motions are less than those for the restricted motions. These differences in the values of the restricted and unrestricted velocities provide the means of testing Dyson's law by this method. It should be possible to assign to each galactic zone a value of κ such that the values of $h\mathrm{V}_1'$ and $h\mathrm{V}_2'$ for $\mu \geqslant 4''$ found for each region in the zone correspond to the values of $h\mathrm{V}_1$ and $h\mathrm{V}_2$.

The following table, giving $h\mathrm{V}'$ for different values of k and $h\mathrm{V}$, is derived from a paper by Dr. Smart * :—

TABLE II.

Values of $h\mathrm{V}^1$.

		$h\mathrm{V}$ 0·0	0·5	1·0	1·5
k	4·0	0·0	0·60	1·10	1·58
	2·0	0·0	0·66	1·22	1·65
	1·0	0·0	0·68	1·25	1·71
	0·0	0·0	0·70	1·26	1·73

* W. M. Smart, *M.N.*, **89**, 98, 1928.

Comparison of Tables I. and II. shows that k decreases from about 4 in Zone IV. to about 2·5 in Zone III. Zone II. may be considered to correspond roughly to $k = 0$, while it will be noticed that the observed differences between $h\mathrm{V}'$ and $h\mathrm{V}$ in Zone I. are too great to be accounted for on the basis of this theory.

One disadvantage of this method is that in the analysis of the drift curves the values of the drift velocities can only be determined with a probable error of approximately 0·1, and if the difference between the restricted and unrestricted drift velocities varies by this amount, the value of k undergoes considerable alterations.

The average number of stars in each region is about 350 for unrestricted proper motions of which about 180 have $\mu \geqslant 4''$, and this is not enough to give us drift curves sufficiently smooth to ensure accurate determination of the elements of the drift. There can be no doubt that method (*b*) would have afforded a more conclusive test of the theory had the data at our disposal been more extensive.

4. *Method* (*c*).—In this method of testing Dyson's law we are concerned with the number of stars in each region whose proper motions exceed a certain quantity. Statistics relating to regions having the same galactic latitude, and for which the values of the drift-velocities were approximately the same, were grouped together so as to give smoother observed curves. This procedure, as we have stated, was only followed in regions having the same galactic latitude, previous investigations besides the results already found by method (*b*) having shown that the quantity k in Dyson's formula is a function of the galactic latitude, increasing from the galactic equator to the poles, in accordance with the known spheroidal distribution of the stars.

An attempt has been made to determine the influence of the errors of observation on the form of $\mathrm{R}(\mu)$, the function we have hitherto used to denote the number of stars in a region whose proper motions are equal to or exceed μ'' per century, but which we shall now for convenience define as the probability of any star having an *observed* proper motion greater than or equal to μ'' per century. We also define $\mathrm{S}(\mu)$ as the probability of any star having a *true* proper motion greater than or equal to μ.

We do not measure a star's proper motion directly, but derive it as the resultant of its rectangular components. Consequently, although we find that the errors of observation of its components are distributed according to the Gaussian law, the resultant errors in the total proper motion, μ, are not in general similarly distributed. When, however, we confine our attention to large values of μ, we may prove that the errors follow a Gaussian law, and on this assumption we shall now proceed to investigate the relation existing between $\mathrm{R}(\mu)$ and $\mathrm{S}(\mu)$, the function defined above. A full discussion of the general case applicable to all values of μ will be given later.

Let the probability of any star having an *observed* proper motion between μ and $\mu + d\mu$ be $u(\mu) \, . \, d\mu$ and let $v(\mu) \, . \, d\mu$ be the corresponding probability function for *true* proper motions.

Then
$$R(\mu) = \int_{\mu}^{\infty} u(\mu) \, . \, d\mu$$
and
$$S(\mu) = \int_{\mu}^{\infty} v(\mu) \, . \, d\mu \qquad . \quad . \quad . \quad (8)$$

We assume that the errors of observation in μ are distributed according to the law
$$\frac{h}{\sqrt{\pi}} \, . \, e^{-h^2\mu^2} \, . \, d\mu$$
where $\frac{\cdot 477}{h}$ = probable error of observation.

Neglecting differences of the third and higher orders, it has been shown that *
$$v(\mu) = u(\mu) - \frac{1}{4h^2} \, . \, u''(\mu) \qquad . \quad . \quad . \quad (9)$$

From (8) we have
$$S(\mu) - R(\mu) = \int_{\mu}^{\infty} \{v(\mu) - u(\mu)\} \, . \, d\mu$$
$$= \int_{\mu}^{\infty} - \frac{1}{4h^2} \, . \, u''(\mu) \, . \, d\mu \text{ by (9)}$$
$$= \frac{u'(\mu)}{4h^2} \quad \text{as} \quad u'(\infty) = 0 \qquad . \quad . \quad (10)$$

Differentiating (8) we have
$$R'(\mu) = - u(\mu)$$
$$R''(\mu) = - u'(\mu).$$

Substituting in (10) we obtain
$$S(\mu) = R(\mu) - \frac{1}{4h^2} \, . \, R''(\mu) \qquad . \quad . \quad . \quad (11)$$

As $R''(\mu)$ is generally positive, it follows that for most values of μ $S(\mu)$ is less than $R(\mu)$; the difference, however, between the true and observed distribution in the range of large values of μ to which the formula (11) is applicable rarely amounts to more than 1 per cent. of $R(\mu)$ and for practical purposes may be neglected.

5. We now proceed to discuss the effect of errors on $R(\mu)$ when μ may take small values. Let the probability of any star having *observed* proper motion components between x and $x + dx$, y and $y + dy$ be
$$u(x, y) \, . \, dx \, . \, dy.$$

Let the corresponding probability for *true* proper motions be
$$v(x, y) \, . \, dx \, . \, dy.$$

* See A. S. Eddington, *M.N.*, **73**, 359, 1913.

Let the errors in the x and y components be distributed according to the law

$$\frac{h}{\sqrt{\pi}} \cdot e^{-h^2\epsilon^2} \cdot d\epsilon.$$

Then it has been shown that *

$$v(x, y) = e^{-\frac{1}{4h^2}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)} \cdot u(x, y).$$

Transforming to polar co-ordinates and expanding,

$$v(\mu, \theta) = u(\mu, \theta) - \frac{1}{4h^2} \cdot \left\{\frac{1}{\mu} \cdot \frac{\partial u}{\partial \mu}(\mu, \theta) + \frac{\partial^2 u}{\partial \mu^2}(\mu, \theta) + \frac{1}{\mu^2} \cdot \frac{\partial^2 u}{\partial \theta^2}(\mu, \theta)\right\}$$

$$+ \frac{1}{2} \cdot \left(\frac{1}{4h^2}\right)^2 \cdot \frac{1}{\mu^2} \cdot \frac{\partial^2 u}{\partial \mu^2}(\mu, \theta) + \text{derivatives of higher order} \qquad (12)$$

The distribution $u(\mu) \cdot d\mu$ and $v(\mu) \cdot d\mu$ of the observed and true total proper motions may now be derived by integrating this equation with respect to θ,

$$u(\mu) \cdot d\mu = \frac{d\mu}{2\pi} \int_0^{2\pi} u(\mu, \theta) \cdot d\theta$$

and

$$v(\mu) \cdot d\mu = \frac{d\mu}{2\pi} \int_0^{2\pi} v(\mu, \theta) \cdot d\theta.$$

Integrating (12) with respect to θ we obtain

$$v(\mu) = u(\mu) - \frac{1}{4h^2} \cdot \left\{\frac{1}{\mu} \cdot \frac{du}{d\mu} + \frac{d^2u}{d\mu^2}\right\} + \frac{1}{32 \cdot h^2} \cdot \frac{1}{\mu^2} \frac{d^2u}{d\mu^2} \cdot \qquad (13)$$

neglecting derivatives of higher order.

Subtracting equation (8)

$$S(\mu) - R(\mu) = \int_\mu^\infty \{v(\mu) - u(\mu)\} \cdot d\mu$$

$$= \int_\mu^\infty \left\{ - \frac{1}{4h^2} \cdot \left(\frac{1}{\mu} \cdot \frac{du}{d\mu} + \frac{d^2u}{d\mu^2}\right) + \frac{1}{32 \cdot h^2} \cdot \frac{1}{\mu^2} \frac{d^2u}{d\mu^2}\right\} \cdot d\mu \qquad (14)$$

The quantity

$$- \frac{1}{4h^2} \cdot \frac{d^2u}{d\mu^2}$$

in the integrand yields as in (10) a term

$$- \frac{1}{4h^2} R''(\mu).$$

* This relation and equation (12) below were given by Dr. Smart, *M.N.*, **88**, 578, 1928.

The two remaining terms in (14) have to be integrated by approximate methods; the integral

$$\int_{\mu}^{u} \frac{1}{\mu} \cdot \frac{du}{d\mu} \cdot d\mu$$

may be made to converge more rapidly if we express it in the form

$$\left| \frac{u}{\mu} \right|_{\mu}^{\infty} + \int_{\mu}^{\infty} \frac{u}{\mu^2} \cdot d\mu$$

$$= \frac{R'(\mu)}{\mu} - \int_{\mu}^{\infty} \frac{R'(\mu)}{\mu^2} \cdot d\mu.$$

Hence (14) reduces to

$$S(\mu) - R(\mu) = -\frac{1}{4h^2} \cdot R''(\mu) - \frac{1}{4h^2} \cdot \frac{R'(\mu)}{\mu} + \frac{1}{4h^2} \int_{\mu}^{\infty} \frac{R'(\mu)}{\mu^2} \cdot d\mu - \frac{1}{32 \cdot h^4} \cdot \int_{\mu}^{\infty} \frac{R'''(\mu)}{\mu^2} \cdot d\mu \quad (15)$$

It will be noticed that for large values of μ this is equivalent to (11). The last term in (15) is always so small that it may be neglected, but for small μ the quantity

$$-\frac{1}{4h^2} \left\{ \frac{R'(\mu)}{\mu} - \int_{\mu}^{\infty} \frac{R'(\mu)}{\mu^2} \cdot d\mu \right\}$$

becomes appreciable. It is quite simple to evaluate the right-hand member of (15); the approximate value of the integral

$$\int_{\mu}^{\infty} \frac{R'(\mu)}{\mu^2} \cdot d\mu$$

is easily found at successively decreasing values of μ. Assuming that the probable error in each component of the proper motion is $\frac{1}{2}''$, the maximum value of the correction to be applied at any point of one of the curves was found to be under 2·5 per cent. of the value of $R(\mu)$ at that point; usually it is much less, and in general the influence of errors is entirely negligible.

6. On assuming a value of k we can construct the curve given by (7) for any region

$$R(\mu) = N_1 \cdot \{1 - \eta^2 \cdot e^{-h^2 V_1^2 (1-\eta^2)}\} + N_2 \cdot \{1 - \eta^2 e^{-h^2 V_2^2 (1-\eta^2)}\}.$$

The curves given by this equation were made to fit the observed statistics for $\mu = 3''$, and that value of k was chosen for each region which gave the closest agreement between the two curves. In practically all of the seventeen regions it was found that, although equation (7) gave a very good representation of the observed facts from $\mu = 3''$ to about $\mu = 7''$, from the latter point onwards the theoretical curves showed a distinct tendency to fall more rapidly than the observed value of $R(\mu)$. Fig. 3 shows the curves for regions Q and R combined,

the small circles representing the observed statistics, while the values of $R(\mu)$ given by (7) are drawn in full. The following table contains

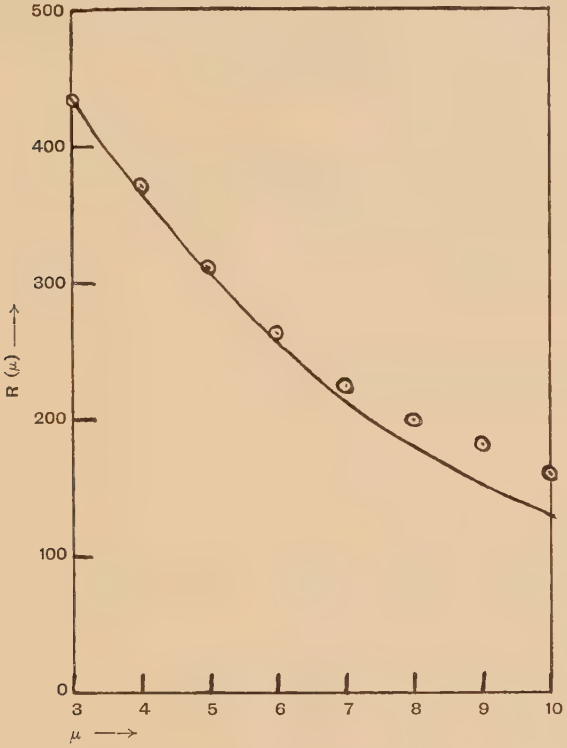

FIG. 3.—Observed and Theoretical Values of $R(\mu)$ for Regions Q and R combined.
$[hV_1 = 1{\cdot}0 \quad hV_2 = 0{\cdot}8 \quad N_1 : N_2 = 10 : 7]$

the values of k found by this method, the closeness of fit between the points $\mu = 3''$ and $\mu = 7''$ being, in general, the criterion directing our choice.

TABLE III.

Values of k.

Region.	k.	Region.	k.
Zone I.		Zone III.	
H	2·0	B and C	3·5
I and K	2·7	D and P	4·5
J	2·7	Q and R	4·0
Zone II.		Zone IV.	
E, G, and L	2·7	A and S	5·0
M	4·0		
N	3·5		

It was impossible to analyse the statistics of region F owing to uncertainties in the values of the drift velocities.

Table III. shows that k increases from Zone I. to Zone IV., taking the values 2·5, 3·2, 4·0, and 5·0 approximately in each zone. This steady increase in the value of k as we proceed away from the galactic equator is a satisfactory feature of the results of this method of testing Dyson's law.

7. An attempt has been made to find an empirical formula for the function $R(\mu)$ which would give a good representation of the form of the observed curves over a larger range of values of μ than that obtained on the basis of Dyson's law. It was found that good agreement with the observed statistics was given by the form

$$R(\mu) = \frac{A}{\mu^n} e^{-k\mu}$$

where A, n, k are adjustable constants.

The curves were made to fit at $\mu = 4''$ and $\mu = 20''$, and the following table shows the values obtained for n and k in the various regions.

Table IV.

	Region.	n.	k.		Region.	n.	k.
Zone I.	H	1·2	0·0	Zone III.	B and C	0·3	0·088
	I and K	0·3	0·089		D and P	0·5	0·055
	J	0·9	0·016		Q and R	1·0	0·0
Zone II.	E, G, and L	0·6	0·06				
	M	0·9	0·027	Zone IV.	A and S	0·5	0·047
	N	0·5	0·058				

There appears to be no correlation between n or k and the mean values of the drift velocities in each region.

8. *Conclusion.*—Both methods (*b*) and (*c*) offer some support for Dyson's law, but it seems that the evidence obtained from the results of (*b*) is not in complete accord with that from (*c*). The principal discrepancy is in the range of values of k demanded by the two methods. From (*b*) we find that if k lies between 0 and 4 the observed statistics are fairly well represented, while method (*c*) requires a range of k from 2·5 to 5. It is probable that more reliance is to be placed upon the results of (*c*) owing to the difficulty, already mentioned, of finding the drift velocities in each region for method (*b*) with a probable error of less than 0·1.

As we have stated previously, the results of (*c*) may be considered as being quite satisfactory. Table III. shows not only that the mean value for k in each zone increases steadily as we move away from the galactic equator, but also that the values of k for the separate regions in each zone do not differ to any great extent from their mean value for that zone.

It is possible that Boss's Catalogue does not contain a sufficient number of stars with small proper motions, and that it cannot on this account be considered as giving us a representative sample of the stars in general. This manifests itself in the portions of the curves of $R(\mu)$ for $\mu = 2''$ and $\mu = 3''$, these appearing in a number of cases not to be sufficiently steep to afford a natural extension of the remaining parts of the curve.

Dr. Smart, using photographic proper motions of stars in regions near the galactic plane, found a value for k about 0·3 or 0·4 from method (*c*). This smaller value for k—our value from this method near the galactic plane is 2·5—corresponds to a distribution of the stars less concentrated than that obtained from a study of meridian proper motions in which it is possible that stars are selected on account of large motion—that is to say, in virtue of the fact that they are comparatively close to the observer.

Note on the Motion of the Perihelion of Mercury.

By J. K. Fotheringham, M.A., D.Litt.

In *Astronomische Nachrichten*, **241**, 105–112, 1931, von Gleich has re-examined Le Verrier's discussion of the transits of Mercury in which he deduced for the motion of the perihelion of Mercury a value exceeding by $+ 38''$ or $+ 39''$ per century its theoretical value. Von Gleich emphasizes the probable errors in the results given by the different transits and shows also how a small difference in the selection of transits for discussion greatly affects the deduced motion of the perihelion. He introduces corrections to the observed time based on the well-known sine term which Newcomb invented to express the great inequality which he had discovered in the Moon's longitude. He gives this term the form which I gave it in my "Solution of Ancient Eclipses of the Sun," * and assumes that the inequality is due entirely to a fluctuation in the Earth's rotation without any compensatory fluctuation in the Moon's motion, but he finds that this fluctuation does not greatly affect the result.† Finally he shows that the uncertainties in the determination of the motion of the perihelion are such as would be produced by an error in the tabular longitude, and he shows that they are of the order that might be produced by the progressive change in the tabular longitude as we pass from one transit to another, if we introduce into the motion of Mercury a secular acceleration bearing the same proportion to the mean motion of Mercury that the secular acceleration of the Sun determined by me in the paper mentioned above bears to the

* *M.N.*, **81**, 125, 1920.

† Now that Dr. de Sitter has found this term in the satellites of Jupiter, I think it certain that it consists of a fluctuation in the Earth's rotation accompanied by a fluctuation in the Moon's motion. There are difficulties in this view, but there are difficulties in every view.

mean motion of the Sun.* This ratio would, of course, hold if the apparent acceleration were, as is generally supposed to be the case, due entirely to a retardation of the Earth's rotation.

Having found that acceleration might produce an error of the right order to annihilate the supposed excess of the observed over the theoretical motion of the perihelion, he does not actually introduce an acceleration term into the discussion, but concludes without further argument that the supposed excess of motion does not exist and that the Newtonian theory of gravitation needs no correction by relativity.

A momentary glance at the equations on which Le Verrier's momentous conclusion depended will show that there is much force in this contention.

Le Verrier† used the two equations—

From November transits :

$$1{\cdot}492\delta n - 1{\cdot}044e' - 0{\cdot}492\varpi' = -0''{\cdot}0310 \quad . \quad (1)$$

From May transits :

$$0{\cdot}712\delta n + 0{\cdot}916e' + 0{\cdot}284\varpi' = +0''{\cdot}1884 \quad . \quad (2)$$

where δn is the correction to the tabular annual mean motion of Mercury in longitude ; e' is the correction to the theoretical annual change in the eccentricity of the orbit, and ϖ' is the correction to the theoretical value obtained by Le Verrier for the annual motion of the perihelion. In solving these equations Le Verrier treated e' as zero.

Now this solution assumes that δn is a constant and that it has the same value for both series of transits. Unless it has the same mean value in both series, the variation in δn will disturb the value of ϖ'.

But an inspection of the transits discussed by Le Verrier will show that, while the mean epoch of the November transits is 1765, the mean epoch of the May transits is 1804. If the Sun's acceleration is $+1''{\cdot}50$ per century, as assumed, then that of Mercury should be $+6''{\cdot}23$, and in forty years the annual motion of Mercury, expressed by δn, should increase by $0''{\cdot}050$. If then δn is to represent the correction to the mean motion at epoch 1765, it will be necessary in the second equation to substitute $\delta n + 0''{\cdot}050$ for δn. That equation then becomes

$$0{\cdot}712\delta n + 0{\cdot}916e' + 0{\cdot}284\varpi' = +0''{\cdot}1528.$$

Putting $e' = 0$ with Le Verrier and eliminating δn, we get $\varpi' = +0''{\cdot}323$ instead of Le Verrier's value $+0''{\cdot}392$.

Von Gleich cites Grossmann‡ for the remark that Le Verrier's theoretical motion of the perihelion of Mercury requires a theoretical correction of $+0''{\cdot}067$. Newcomb found the same. If we deduct this from the total correction just obtained, the empirical correction sinks to $+25''{\cdot}6$ per century, contrasted with Einstein's relativity

* Von Gleich does not refer to my paper. Both these terms are obtained from Meyermann, *Ergebnisse der exakten Naturwissenschaften*, **7**, 96, 97, 1928.

† *Annales de l'Observatoire imperial de Paris*, Tome v. 77, 78, 1859.

‡ *Astronomische Nachrichten*, **214**, 41–54, 195, 1921.

correction of + 42″·9. If the result thus obtained were really consistent with the evidence, it is clear that the observed motion of the perihelion of Mercury would not support the theory of relativity.

This solution of mine has, however, ignored the effect of fluctuation which cannot so easily be given a mean value, but must be introduced separately into each equation of condition. If von Gleich is right in supposing that this effect is small, his criticism is fully justified so far as the evidence contained in Le Verrier's discussion goes. But a far longer series of transits extending to 1924 has been discussed and most carefully reduced by Dr. Innes,* and Dr. de Sitter has made these the basis of a discussion in which the accelerations and fluctuations are determined, but in which no assumption is made as to the motion of the perihelion of Mercury.† It is, however, a very easy exercise to deduce it from the corrections found by Dr. de Sitter.

He gives the two equations—

$$\delta V = - ''\cdot 55 - ''\cdot 42T$$
$$\delta W = + ''\cdot 53 + ''\cdot 44T,$$

where δV contains certain corrections applicable to the November transits and δW the corresponding corrections applicable to the May transits. Fully expressed,

$$\delta V = - 0\cdot 487\delta\pi_1 - 1\cdot 137\delta e_1 + 0\cdot 0200\delta\pi + 1\cdot 58\delta e - 1\cdot 01\delta L'$$
$$\delta W = + 0\cdot 284\delta\pi_1 + 0\cdot 896\delta e_1 - 0\cdot 0186\delta\pi - 1\cdot 62\delta e - 0\cdot 97\delta L'$$

where $\delta\pi_1$, δe_1 are the corrections to the longitude of the perihelion and to the eccentricity of Mercury, and $\delta\pi$, δe the corresponding terms in relation to the Earth, and $\delta L'$ is the correction to the Sun's mean longitude.

On these equations Dr. de Sitter remarks: "We can add an arbitrary quantity z to $\delta\lambda_1$ (the correction to the mean longitude of Mercury), if we subtract at the same time $1\cdot 487z$ from δV and $0\cdot 716z$ from δW."

Confining our attention to the annual changes and ignoring the corrections to the Sun's longitude and to the eccentricities, and putting π' for the correction to the annual motion of the perihelion of the Earth, we obtain the following equations analogous to (1) and (2) above.

From November transits:

$$1\cdot 487\delta n - 0\cdot 487\varpi' + 0\cdot 0200\pi' = - 0''\cdot 0042$$

From May transits:

$$0\cdot 716\delta n + 0\cdot 284\varpi' - 0\cdot 0186\pi' = + 0''\cdot 0044.$$

If in a first solution we put $\pi' = 0$ and eliminate δn, we get $\varpi' = + ''\cdot 0123$ or, expressed in centuries, $+ 1''\cdot 23T$.

This correction has to be applied to Newcomb's value of the motion

* *Circular No. 65 of the Union Observatory of South Africa*, 305–324, 1925.

† *Bulletin of the Astronomical Institutes of the Netherlands*, No. 124, 28–31, 1927.

of the perihelion of Mercury, which Newcomb believed to be 43″·37 per century in excess of the theoretical value.* We thus obtain as the excess of the observed over the theoretical motion, 44″·60 per century. This is so close to Einstein's relativity term of 42″·9 per century that it seems reasonable to complete the solution by substituting the Einstein value of the motion of the Earth's perihelion for that which Newcomb had deduced from Asaph Hall's theory. This means that we put $\pi' = -0''{\cdot}066$. ϖ' now becomes $= +0''{\cdot}0088$, and the excess of the observed over the theoretical motion of the perihelion of Mercury becomes $44''{\cdot}25T$. Reduced to the equinox of Newcomb's theory, this is 43″·95 per century.

It is of no use to be more precise. The theoretical value will be affected by small errors in the adopted masses of the planets, especially of Venus, and the observed value by possible errors in the assumed changes in the eccentricities, in the assumed terms in the Earth's motion, in the assumed mean of the stars, and in the adopted values of the accelerations and fluctuations. It is, therefore, impossible to say on which side the error, if any, in the Einstein term lies.

The reason why the large corrections for acceleration and fluctuation which Le Verrier's discussion requires improve instead of destroying the agreement of observation with the theory of relativity is that, owing to the distribution of the observations, the two corrections tell in opposite directions, the correction for fluctuation being the larger. This is more or less an accident. If von Gleich has failed to destroy the theory of relativity, he has at least done a service by showing to how large an extent the determination of constants in the solar system may be affected by acceleration and fluctuation. In fact it has become necessary to determine over again all constants depending on the time.

University Observatory, Oxford:
1931 *September* 9.

Mean Areas and Heliographic Latitudes of Sunspots in the Year 1930.

(*Communicated by the Astronomer Royal.*)

The results here given are in continuation of those in the *Monthly Notices*, **91**, 271, 1931, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at the Kodaikanal Observatory. Photographs for August 23, December 23 and 24 were kindly lent for measurement by the Ebro Observatory, Tortosa, Spain.

Table I. gives the mean daily areas of umbræ, whole spots, and faculæ for each synodic rotation of the Sun included in the year 1930, whilst the mean values for each year as a whole from 1923 to 1930 are given in Table II.

* *Astronomical Constants*, 184, 1895.

TABLE I.

No. of Rotation.	Rotation commenced G.C.T.	No. of Days on which Sun was Photographed.	Mean of Daily Areas. Projected.* Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening.† Umbræ.	Whole Spots.	Faculæ.
1020	Dec. 15·41	27	560	2830	3002	406	2056	3564
1021	Jan. 11·74	27	335	1586	2837	247	1200	3338
1022	Feb. 8·09	28	172	890	2544	127	643	2984
1023	Mar. 7·42	27	163	795	1882	118	584	2202
1024	Apr. 3·72	27	148	708	1522	106	497	1830
1025	Apr. 30·98	27	151	695	1404	107	484	1648
1026	May 28·20	27	112	503	1047	79	354	1256
1027	Jun. 24·40	28	56	214	820	42	160	991
1028	Jul. 21·61	27	56	271	749	41	210	907
1029	Aug. 17·83	27	133	613	945	98	452	1139
1030	Sept. 14·08	27	122	688	911	86	499	1085
1031	Oct. 11·36	28	141	722	756	100	514	893
1032	Nov. 7·66	27	177	936	1058	127	673	1251
1033	Dec. 4·97	27	62	263	890	47	204	1038

TABLE II.

Year.	No. of Days. Photographed.	Without Spots.	Mean Daily Areas. Projected.* Umbræ.	Whole Spots.	Faculæ.	Corrected for Foreshortening.† Umbræ.	Whole Spots.	Faculæ.
1923	365	171	15	74	189	10	55	222
1924	366	97	62	361	498	46	276	575
1925	365	19	189	1102	1508	141	830	1750
1926	365	2	284	1651	2159	217	1262	2556
1927	365	0	248	1417	1812	186	1058	2212
1928	366	0	333	1884	2134	244	1390	2589
1929	365	0	334	1695	2190	245	1242	2567
1930	365	4	145	705	1378	106	516	1630

Table III. gives for each rotation in the year 1930 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for both hemispheres, northern and southern. The mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots are also tabulated. The mean values for the years 1923 to 1930 are given in Table IV. Tables II. and IV. are in continuation of similar tables given in the *Monthly Notices*: for the years 1874 to 1888, **49,** 381 and 382; 1889

* Expressed in millionths of the Sun's disc.

† Expressed in millionths of the Sun's hemisphere.

to 1902, **63,** 465 and 466; 1901 to 1914, **76,** 402 and 403; 1913 to 1924, **85,** 1007 and 1008.

TABLE III.

No. of Rotation.	Rotation commenced G.C.T.	Spots NORTH of the Equator. Mean Daily Area.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean Daily Area.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1020	Dec. 15·41	1569	10·28	487	4·50	+6·78	8·91
1021	Jan. 11·74	861	11·74	339	8·61	+5·99	10·85
1022	Feb. 8·09	126	11·89	517	14·43	−9·26	13·93
1023	Mar. 7·42	246	10·67	337	10·72	−1·69	10·70
1024	Apr. 3·72	303	12·76	194	7·19	+4·97	10·58
1025	Apr. 30·98	301	14·20	182	9·21	+5·38	12·32
1026	May 28·20	241	15·11	113	6·70	+8·15	12·43
1027	Jun. 24·40	88	9·02	72	6·34	+2·08	7·81
1028	Jul. 21·61	50	2·89	160	6·80	−4·48	5·87
1029	Aug. 17·83	330	10·51	122	9·32	+5·16	10·19
1030	Sept. 14·08	420	7·96	79	4·00	+6·07	7·33
1031	Oct. 11·36	202	8·78	312	6·03	−0·20	7·11
1032	Nov. 7·66	284	6·42	389	8·11	−1·98	7·40
1033	Dec. 4·97	128	9·98	76	10·29	+2·44	10·10

TABLE IV.

Year.	No. of Days on which Sun Photographed.	Spots NORTH of the Equator. Mean Daily Area.	Spots NORTH of the Equator. Mean Heliographic Latitude.	Spots SOUTH of the Equator. Mean Daily Area.	Spots SOUTH of the Equator. Mean Heliographic Latitude.	Mean Latitude of entire Spotted Area.	Mean Distance from Equator of all Spots.
			°		°	°	°
1923	365	33	13·88	22	17·32	+ 1·34	15·26
1924	366	232	22·19	45	25·54	+14·49	22·73
1925	365	517	20·70	313	19·37	+ 5·60	20·20
1926	365	663	19·27	599	17·98	+ 1·60	18·66
1927	365	379	17·75	679	13·54	− 2·33	15·05
1928	366	727	11·93	663	15·23	− 1·02	13·50
1929	365	656	10·73	586	10·27	+ 0·82	10·51
1930	365	286	10·60	230	8·96	+ 1·88	9·87

The rotations in Tables I. and III. are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node of the Sun's equator on the ecliptic at mean noon on 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

The principal features of the record for the year 1930 are as follows :—

1. A marked decline of solar activity shown by a decrease in (*a*) the mean daily area of sunspots ; (*b*) the number of groups of spots ; (*c*) the mean daily area of faculæ, the values of which were $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{3}{5}$ respectively those of the preceding year.

2. On four days no spots were seen on the Sun's disc. On one day no faculæ were measured.

3. The northern hemisphere was slightly the more active.

4. The mean latitude of spots (9°·9) was nearly the same as in 1929. Latitudes ranged from 0° to 27°.

5. The number of groups lasting for two days or more was 192 ; of these 104 were in the northern hemisphere and 88 in the southern. Besides these, 84 very small groups (usually consisting of one or two tiny spots) were measured on one day only ; 44 were in the northern hemisphere and 40 in the southern.

6. An inspection of Table I. shows a rapid falling off in activity during the first half of the year, after which a partial recovery set in. From Tables II. and IV. it is clear that the Sun's activity is advanced in the declining phase of the 11-year cycle.

Royal Observatory, Greenwich :
1931 *August* 31.

Note on Professor Plaskett's Paper : " The Formation of the Magnesium b *Lines in the Solar Atmosphere."* By M. Minnaert and G. F. W. Mulders.

(*Communicated by the Secretaries.*)

The apparently unsatisfactory agreement between the intensity measurements on the *Mg b* line made at Utrecht (*Zs. f. Astrophysik*, **1**, 199, 1930), and those made by Prof. H. H. Plaskett (*M.N.*, **91**, 870, 1931), is due to a very simple mistake. For every intensity measured by us, the corresponding " width " of the line has been given in angstroms. We are very sorry that confusion has arisen from that expression, which Professor Plaskett has interpreted as " *width measured from the centre of the line towards one side.*" An integration over the total area and a comparison with our " total intensity " proves at once that we really meant *the total width.*

By plotting our points together with those of Professor Plaskett, an excellent agreement is found, especially if the amended value of the residual intensity given by us is taken into account.*

As to the ghosts of the Utrecht grating, we refer to *Versl. Akad. Amsterdam*, 1921 September, p. 257, where their intensities have been given. The strongest ghost in the first order has only 0·006 of the intensity of the spectrum line itself, and this may be considered as fairly faint. Moreover, all our measurements have been corrected for that small source of error.

* *Zs. f. Astrophysik*, **2**, 178, 1931.

Observations of Minor Planets and of the Comet Nagata (1931 *b*), *made with the* 243-*mm. Merz-Repsold Refractor at the Astronomical Observatory, Kiew.* By Professor S. D. Tscherny.

(*Communicated by the Secretaries.*)

Amphitrite (29).

Date 1931.	G.M.T.	$\Delta\alpha$.	$\Delta\delta$.	Number of Comparisons.	$\alpha_{1931\cdot0}$.	log $p\Delta$.	$\delta_{1931\cdot0}$.	log $p\Delta$.
	h m s	m s	′ ″		h m s		° ′ ″	
Mar. 22	17 50 25	+0 10·82	−1 50·4	6 : 6	8 33 4·35	8·891*n*	+23 27 35·5	0·601
22	18 8 8	+0 10·80	−1 53·3	5 : 5	8 33 4·33	8·668*n*	+23 27 32·6	0·598
23	17 27 6	+0 13·52	−5 26·9	5 : 5	8 33 7·05	9·046*n*	+23 23 59·0	0·607
23	17 49 5	+0 13·82	−5 30·6	5 : 5	8 33 7·35	8·861*n*	+23 23 55·3	0·601

Flora (8)

June 6	21 59 0	−5 54·23	+1 26·8	3 : 3	16 38 10·00	8·239	−15 48 54·7	0·905
16	21 4 6	−1 16·67	−0 2·8	6 : 6	16 27 25·34	8·256	−15 50 19·2	0·905
17	19 57 31	−2 14·70	−0 31·3	6 : 6	16 26 27·31	8·909*n*	−15 50 47·7	0·903
18	19 52 44	−3 14·27	−1 5·2	6 : 6	16 25 27·74	8·927*n*	−15 51 21·6	0·903
20	20 23 18	−5 12·37	−2 27·0	3 : 3	16 23 29·64	8·017*n*	−15 52 43·4	0·907
22	20 4 20	−7 3·95	−4 7·2	4 : 4	16 21 38·06	8·432*n*	−15 54 23·6	0·905
22	20 34 15	−0 2·10	+1 10·4	5 : 5	16 21 36·84	8·409	−15 54 31·2	0·905
23	19 31 25	−0 53·46	+0 15·1	5 : 5	16 20 45·48	8·862*n*	−15 55 26·5	0·904

Mnemosyne (57).

Aug. 5	19 1 16	−1 24·62	−1 19·8	3 : 3	19 47 36·52	9·247*n*	+0 26 56·6	0·827
6	18 54 37	−2 5·69	−6 24·9	3 : 3	19 46 55·45	9·250*n*	+0 21 51·5	0·827

Interamnia (704).

Aug. 13	19 53 46	+1 18·95	+6 9·1	5 : 5	22 5 34·04	9·402*n*	+10 31 8·1	0·876
22	20 17 25	−0 43·33	+4 6·7	5 : 5	21 57 54·66	9·201*n*	+10 42 32·5	0·754
Sept. 1	19 57 15	−2 13·40	+2 55·0	3 : 3	21 49 5·15	9·062*n*	+10 39 39·1	0·751
4	19 0 36	−1 17·48	−0 21·0	5 : 5	21 46 38·02	9·255*n*	+10 34 23·8	0·758
4	19 24 14	−1 18·32	−0 22·4	5 : 5	21 46 37·18	9·162*n*	+10 34 22·4	0·754
5	19 36 40	−2 6·96	−2 32·5	7 : 7	21 45 48·54	9·067*n*	+10 32 12·3	0·753
6	18 8 40	−1 37·86	+0 59·4	5 : 5	21 45 4·07	9·379*n*	+10 30 0·7	0·766
6	18 30 20	−1 38·92	+0 57·3	4 : 4	21 45 3·01	9·321*n*	+10 29 58·6	0·762
6	18 53 50	−1 39·30	+0 55·4	4 : 4	21 45 2·63	9·248*n*	+10 29 56·7	0·758
7	19 34 9	−2 27·82	−1 35·5	5 : 5	21 44 14·11	9·024*n*	+10 27 25·8	0·753
7	19 59 22	−2 28·60	−1 37·8	5 : 5	21 44 13·33	8·813*n*	+10 27 23·5	0·751

Comet 1931 *b*.

Sept. 13	18 3 18	−0 34·56	+6 46·8	2 : 2	13 56 25·24	9·577	+8 15 23·7	0·819
14	17 58 56	−0 7·15	−2 20·1	4 : 4	..	9·577	..	0·819

Mean Positions of the Comparison Stars.

No.	Mag.	$\alpha_{1931\cdot0}$	$\delta_{1931\cdot0}$	Catalogue.
		h m s	° ′ ″	
1	8·6	8 32 53·53	+23 29 25·9	Berlin B 3449, Küstner 3802
2	8·2	16 44 4·23	−15 50 21·5	Washington 6024
3	8·2	16 28 42·01	−15 50 16·4	Washington 5963
4	8·9	16 21 38·94	−15 55 41·6	Washington 5938
5	8·8	19 49 1·14	+ 0 28 16·4	Nicolajew 5002
6	8·1	22 4 15·09	+10 24 59·0	Leipzig I 8830
7	7·1	21 58 37·99	+10 38 25·8	Leipzig I 8780
8	8·7	21 51 18·55	+10 36 44·1	Leipzig I 8722
9	8·5	21 47 55·50	+10 34 44·8	Leipzig I 8694, Küstner 9672
10	8·3	21 46 41·93	+10 29 1·3	Leipzig I 8685
11	8·9	13 56 59·80	+ 8 8 36·9	Leipzig II 6570
12	9·5	13 59	+ 8 14	B.D. +8° 2808

Remarks.—Observations were made with the filar micrometer (eyepiece with a magnifying power 145).

1931 June 6. Sky cloudy.
August 22. Image very faint.
September 5. Wind.
September 13, 14. Circular nebulosity with a central condensation. Image very faint. Comet near the horizon. Observations difficult. Illuminated threads.

Astronomical Observatory, Kiew:
1931 *September* 18.

Errata.

p. 482, lines 6–8, Professor Milne's paper, *for* "for the left-hand side is given and the right-hand side can take any value between 0 and ∞," *read* "for the left-hand side is given, and can be shown to lie between 0 and 3, and the right-hand side can take any value between 0 and 3.

p. 562, Mr. Fletcher's paper, insert l before $=$, in equation (15).

p. 816, Professor Pickering's paper, the last footnote should read *Harvard Annals* (1919), **82**, 59; *Annual Report of the Director of the Mount Wilson Observatory*, 1929-30, 160; *Popular Astronomy* (1931), **39**, 2.

LIST OF ADDITIONS

TO THE LIBRARY

OF THE SOCIETY

1930 JUNE 30 TO 1931 JUNE 30.

Serial Publications received by exchange in the regular course, and Reprints of Papers published in some of these Serials, are not included in this Annual List.

An asterisk (*) indicates that the work is an excerpt.

Aarhus, Ole Rømer Observatoriet:
Meddelelser, Nos. 1–5.
(*Observatory.*) 4to. Aarhus, 1925–30

Achelis (Elizabeth):
The World Calendar. Second edition.
(*World Calendar Association Inc.*) 8vo. New York, 1931

American Association of Variable Star Observers:
*Monthly Reports and Annual Report for the year 1930.
(*Association.*) 8vo. Northfield, 1930

American Geophysical Union:
Transactions. Tenth Annual Meeting, 1929 April 25, 26. Eleventh Annual Meeting, 1930 May 1 and 2, Washington, D.C.
(*National Research Council.*)
8vo. Washington, D.C., 1930

American Meteor Society:
Bulletin, No. 11.
(*Dr. C. P. Olivier.*) 4to. Upper Darby, Pa., 1930

Antoniadi (E.-M.):
La planète Mars. Étude basée sur les résultats obtenus avec la grande lunette de l'Observatoire de Meudon et exposé analytique de l'ensemble des travaux exécutés sur cet astre depuis 1659.
(*Purchased.*) 4to. Paris, 1930

Bamberg, Remeis-Sternwarte:
Veröffentlichungen. Band 3.
(*Observatory.*) 4to. Bamberg, 1931
Band 3. E. Zinner und A. A. Wachmann, Die Bearbeitung von Winnecke's Beobachtungen veränderlicher Sterne.

Banachiewicz (Th.):
*Methodes arithmométriques de la correction des orbites (directes et à quaternions).
(*Author.*) 8vo. Cracovie, 1929

Barcelona, Observatorio Fabra:
Boletin, Sección Astronómica, Tomo II, No. 2.
(*Observatory.*) 4to. Barcelona, 1931

Barlow (C. W. C.) and G. H. Bryan:
Elementary Mathematical Astronomy. Fourth edition, revised by A. C. D. Crommelin.
(*Publishers.*) 8vo. Oxford, 1930

Barlow (Peter):
Barlow's Tables of squares, cubes, square roots, cube roots, and reciprocals of all integer numbers up to 10,000. Third edition, edited by L. J. Comrie.
(*Publishers.*) 8vo. London, 1930

Becker (Fr.):
*Über interstellare Massen und die Absorption des Sternlichtes im Weltraum.
(*Author.*) 8vo. Berlin, 1930

Berlin, Preussische Akademie der Wissenschaften:
Geschichte des Fixsternhimmels. Abteilung I, Bände 13, 14, 15.
(*Purchased.*) 4to. Karlsruhe, 1930–31

Band 13. 12^h Rektaszension.
Band 14. 13^h Rektaszension.
Band 15. 14^h Rektaszension.

Bonn, Universitäts-Sternwarte:
Veröffentlichungen. Nos. 21–25.
(*Observatory.*) 4to. Bonn, 1930

Brändli (H.):
Astronomical Data for the year 1930.
(*Author.*) 8vo. [Bangkok], [1930]

Budapest, Hungarian National Observatory [Konkoly]:
Astronomische Abhandlungen, Band I, No. 2.
(*Observatory.*) 8vo. Budapest, 1930

Bulletin Astronomique:
Mémoires et Variétés, deuxième série, $1^{ère}$ partie, Tomes II, III, IV, V.
(*Paris Observatory.*) 8vo. Paris, 1922–29
——: Mémoires et Variétés, deuxième série, Tome VI, Fasc. 1–6.
(*Paris Observatory.*) 8vo. Paris, 1930

Bulletin Astronomique:
Revue générale des travaux astronomiques, 2me série, 2me partie, Tome 2, Tome 3 (except Nov., Dec.), Tome 4 (except Oct., Nov., Dec.), Tome 5, Tome 5 appendice.
(*Paris Observatory.*) 8vo. Paris, 1920–28

Cambridge, Solar Physics Observatory:
Annals, Vol. 2, Pt. 1.
(*Observatory.*) 4to. Cambridge, 1930
Vol. 2, Part 1. F. E. Baxandall, The Spectrum of β Lyræ.

Canada, Geodetic Survey:
Annual Report of the Superintendent . . . for the fiscal year ending March 31, 1929.
(*Superintendent.*) 8vo. Ottawa, 1930

Canberra, Commonwealth Solar Observatory:
Memoirs, Nos. 1, 2.
(*Observatory.*) 4to. Melbourne, 1928–30
No. 1. The Luminosity of the Night Sky.
No. 2. W. B. Rimmer, The Luminosities and Parallaxes of 350 stars of spectral type B.

Canton, Observatoire de l'Université Sun Yat Sen:
Revue Bimensuelle, Tome 1, No. 6; Tome 2, No. 1.
(*Observatory.*) 4to. Canton, 1930–31

Cape of Good Hope, Royal Observatory:
Cape Star-Correction Tables. Tables of $(1+x)$ and $(1+y)$.
(*Observatory.*) 8vo. s.l. [1930]

Catania, Reale Osservatorio Astrofisico:
Catalogo Astrofotografico, 1900·0. Zona $+46°$ e $+55°$. Vol. 5, parte 3 e 4, Vol. 6, parte 3 e 4, Vol. 8, parte 7 e 8.
(*Observatory.*) 4to. Catania e Napoli, 1929–30

Chatley (Herbert):
*The Principles of Rocket Propulsion.
(*Author.*) 8vo. Shanghai, 1931

Cincinnati Observatory:
Publications, No. 20.
(*Observatory.*) 4to. Cincinnati, 1930
No. 20. J. G. Porter, E. I. Yowell, E. S. Smith, A Catalog of 1474 Stars with proper-motion exceeding four-tenths of a second per year.

Clacey, John:
John Clacey, Optician: An Appreciation of his Half-Century of Devotion to the Art of Lens Making. By Members of the Staff of the National Bureau of Standards.
(*Bureau of Standards.*) 8vo. Northfield, 1930

Comendatoff (N.):
*Tables de mouvement de la planète ① Cérès pour les années 1801–2000, préparées d'après la théorie de M. Vilïeff.
(*Author.*) 4to. Leningrad, 1930

Comrie (L. J.):
——: The Hollerith and Powers Tabulating Machines. Being a Lecture delivered under the auspices of the Office Machinery Users' Association at the London School of Economics on 1929 November 20.
(*Author.*) 4to. London, 1930
——: Four-figure Tables of the Natural and Logarithmic Trigonometrical Functions with the Argument in Time.
(*Purchased.*) 8vo. London, 1931
See also **Milne-Thompson and Comrie.**

Corbu (J.):
——: Die Rotation der nicht soliden Körper.
(*Author.*) 8vo. Bistrita, 1926
——: *La rotation non uniforme du soleil, les courants marins, les calmes, les vents alizés et les bandes de Jupiter et de Saturne considérés comme un même phénomène.
(*Author.*) 8vo. Cluj, 1931

Córdoba, Observatorio Nacional Argentino:
Resultados, Vol. 31, 35.
(*Observatory.*) 4to. Córdoba, 1928–29
Vol. 31. Catálogo Astrográfico. Zona −29°.
Vol. 35. Primer Catálogo Fundamental de Córdoba de 761 estrellas para el equinoccio medio de 1900·0.

Crawford (Russell Tracy):
Determination of Orbits of Comets and Asteroids.
(*Purchased.*) 8vo. New York, 1930

Dessau, Naturwissenschaftlicher Verein:
Berichte. Heft 1. Über die Jahre 1924–1928.
(*Society.*) 8vo. Dessau, 1929
[Contains Life of Schwabe].

Dublin, Royal Dublin Society:
Bi-Centenary Souvenir, 1731–1931.
(*Society.*) 4to. Dublin, 1931

Eddington (A. S.):
——: Matter in Interstellar Space. (Broadcast National Lecture, 1929 April 15.)
(*Purchased.*) 8vo. London, 1929
——: The Rotation of the Galaxy; being the Halley Lecture delivered on 1930 May 30.
(*Author.*) 8vo. Oxford, 1930

Encyclopädie der mathematischen Wissenschaften:
Band 6 (2B), Hefte 3, 4.
(*Purchased.*) 8vo. Leipzig, 1930–31

Fortschritte der Himmelskunde seit Kepler. (Süddeutsche Monatshefte, 1930 November, Jahrgang 28, Heft 2.)
(*Purchased.*) 8vo. München, 1930

Gheury de Bray (M. E. J.):
Elementary Hyperbolics for technical and other students. Specially adapted to the requirements of beginners.
(*Author.*) 2 vols. 8vo. London, 1931
Vol. 1. Hyperbolic functions of real and unreal angles.
Vol. 2. The Application of hyperbolic functions.

Guyot (Edmond):
Étude sur l'Instrument des passages et la détermination de l'heure.
(*Observatoire de Genève.*) 8vo. Neuchatel, 1931

Haalck (H.):
——: *Die Verwendung der Elektrizität zur Erforschung des Untergrundes.
(*Author.*) 8vo. Leipzig, 1929
——: *Über das Vorhandensein einer magnetischen Wirkung durch rotierende Massen und die Ursache des Erd- und Sonnenmagnetismus.
(*Author.*) 8vo. Braunschweig, 1930
——: *Zur Frage der Beschaffenheit des Erdinnern.
(*Preussisches Geodätisches Institut.*) 8vo. Braunschweig, 1931

Handbuch der Astrophysik: Herausgegeben von G. Eberhard, A. Kohlschütter, H. Ludendorff. Bände II, III.
(*Purchased.*) 8vo. Berlin, 1930–31
Band II. Zweite Hälfte. Grundlagen der Astrophysik, Zweiter Teil, II. Bearbeitet von G. Eberhard, W. Hassenstein.
Band III. Zweite Hälfte. Grundlagen der Astrophysik, Dritter Teil, II. Bearbeitet von W. Grotian, O. Laporte, E. A. Milne, K. Wurm.

Hellman (C. Doris):
*George Graham. Maker of Horological and Astronomical Instruments.
(*Dr. Caroline E. Furness.*) 8vo. s.l., 1931

Helsingfors, Baltic Geodetic Commission:
Special Publication, No. 1.
(*Commission.*) 8vo. Helsingfors, 1930

Helsingfors, Observatoire:
Catalogue Photographique du Ciel. Tome VIII, fasc. 1.
(*Observatory.*) 4to. Helsingfors, 1930
Tome VIII, fasc. 1. Clichés de 21^h à 22^h 30^m.

Hyderabad, Nizamiah Observatory:
Astrographic Catalogue 1900·0, Part II, zones $-20°$ to $-24°$. Vol. 7, Dec. $-23°$.
(*Observatory.*) 4to. Edinburgh, 1930

International Astronomical Union:
Bulletin for Character Figures of Solar Phenomena. Published by the Eidgenossische Sternwarte in Zürich. Nos. 9–12.
(*I.A.U.*) 4to. Zürich, 1930–31

International Geodetic and Geophysical Union:
——: Section de Météorologie. Quatrième Assemblée générale: Stockholm, Août 1930.
(*I.G. and G. Union.*) 8vo. Paris, 1930
——: Section de Météorologie. Quatrième Assemblée Générale, Stockholm, 14–23 Août 1930. Rapport du Bureau de la Section.
(*I.G. and G. Union.*) 4to. Paris, 1930

Janssen (Jules):
Œuvres Scientifiques recueillies et publiées par Henri Dehérain. Tomes 1, 2.
(*H. Dingle.*) 2 vols. 8vo. Paris, 1929–30

Jeans (Sir James):
——: The Mysterious Universe.
(*Author.*) 8vo. Cambridge, 1930
——: The Stars in their Courses.
(*Purchased.*) 8vo. Cambridge, 1931

Kasan, Observatoire Astronomique de l'Université:
Publications. No. 26.
(*Observatory.*) 8vo. Kasan, 1930
In Russian, with summary in German.

Kiel, Sternwarte:
Publikationen, XVI, XVII.
(*Observatory.*) 4to. Kiel, 1930

Kienle, (H.)
*Das Temperaturproblem in der Astrophysik.
(*Author.*) 8vo. Berlin, 1931

King (E. F.):
A Biographical Sketch of Sir Isaac Newton. To which are added authorised reports of the oration of Lord Brougham (with His Lordship's notes) at the inauguration of the statue at Grantham; and of several of the speeches delivered on that occasion.
(*Prof. Stratton.*) Sm. 8vo. Grantham and London, 1858

King (Edward Skinner):
A Manual of Celestial Photography. Principles and Practice for those interested in photographing the heavens. With foreword by Harlow Shapley.
(*Purchased.*) 8vo. Boston, 1931

Knobel (E. B.):
Drawings of Jupiter, Comet 1881 *b*, Encke's Comet 1871, Paul Henry's Comet 1873, and Borrelly's Comet 1873.
(*Miss Knobel.*) MS. 6 sheets

Leavenworth (F. P.):
Measures of Double Stars.
See Minneapolis, University of Minnesota, Publications.

Lemaître (G.):
——: *Un univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extra-galactiques.
(*Author.*) 8vo. Louvain, 1927
——: *Le mouvement varié d'un solide, d'après la théorie de la relativité.
(*Author.*) 8vo. Louvain, 1927
——: *La grandeur de l'espace. Conférence faite le 31 janvier 1929 à la Société Scientifique de Bruxelles.
(*Author.*) 8vo. Louvain, 1929

Lewis (T.):
*Notes on some historical instruments at the Royal Observatory, Greenwich.
(*Purchased.*) 8vo. London, 1890

Lisbon, Academia das Sciências:
Boletim, Nova Série, Vol. 1.
(*Academy.*) 8vo. Coimbra, 1929

London, Royal Astronomical Society:
George Darwin Lecture, 1931. 8vo. London, 1931
W. de Sitter, Jupiter's Galilean Satellites.

Lund, Astronomiska Observatorium:
——: Annals. No. 2.
(*Observatory.*) 4to. Malmö and Lund, 1931
No. 2. Per Collinder, On structural properties of open galactic clusters and their spatial distribution. With an appendix containing a catalogue of 471 objects.
——: Circular. Nos. 1–3.
(*Observatory.*) 4to. Lund, 1931

Mackaye (James):
The Dynamic Universe.
(*Publishers.*) 8vo. London, 1931

MacPherson (John):
John von Lamont, Astronomer Royal of Bavaria. A Memoir.
(*Purchased.*) Sm. 8vo. Braemar, 1930

Milne-Thomson (L. M.), and L. J. Comrie:
Standard Four-Figure Mathematical Tables, including many new tables, trigonometrical functions for radians, inverse trigonometrical and hyperbolic functions, and an extended table of natural logarithms. Edition A with positive characteristics in the logarithms.
(*Authors.*) 8vo. London, 1931

Minneapolis, University of Minnesota, Astronomical Observatory:
Publications. Vol. 1.
(*Observatory.*) 4to. Minneapolis, 1930
Vol. 1. Measures of Double Stars by Francis P. Leavenworth, with which are included the measures by William O. Beal.

Naples, R. Osservatorio Astronomico di Capodimonte:
——: Rapporto sull'attività dell'Osservatorio di Capodimonte nel 1929
(*Observatory.*) 8vo. Napoli, 1929
——: *L'Observatoire de Capodimonte à Naples, par E. Leroy et Thérèse Leroy.
(*Observatory.*) 8vo. Paris, 1929

Neumann (E. M. S. Navarro):
——: *Un petit grain blanc, enregistré par des séismographes.
(*Author.*) 8vo. Braunschweig, 1931
——: *Notas sismológicas del año 1930.
(*Author.*) Royal 8vo. Barcelona, 1931

Newton (Isaac):
——: A New and Most Accurate Theory of the Moon's Motion; whereby all her Irregularities may be solved, and her Place truly calculated to Two Minutes. Written by that Incomparable Mathematician, Mr. Isaac Newton, and Published in *Latin* by Mr. David Gregory in his *Excellent Astronomy.*
(*Purchased.*) Sm. 8vo. London, 1702
See also **King (E. F.)**

Nikolajew Astronomical Observatory:
Publications, No. 1.
(*Observatory.*) 8vo. Nikolajew, 1930

Palisa (Johann) und Max Wolf:
Photographische Sternkarten (Aequin. 1875·0). Serie XI. (10 charts.)
(*Purchased.*) Wien, 1931

Paris, Institut International de Co-opération Intellectuelle:
Co-ordination des Bibliothèques. Guide des services nationaux de renseignements du prêt et des échanges internationaux. (*Joint Standing Committee on Library Co-operation.*)
8vo. Paris, 1930

Paris, Observatoire:
Catalogue photographique du Ciel. Co-ordonnées rectilignes. Tome 6. Zone +18° à +20°.
(*Observatory.*) 4to. Paris, 1929

Payne (Cecilia H.):
The Stars of High Luminosity. Harvard Observatory Monograph, No. 3.
(*Radcliffe College, Harvard University.*) 8vo. New York, 1930

Philadelphia, The Rittenhouse Astronomical Society:
Leaflet No. 1. April 1931.
(*Prof. J. H. Pitman.*) Sm. 8vo. Philadelphia, 1931
No. 1. Maurice J. Babb, Who was David Rittenhouse ?

Poznań, Observatoire Astronomique de l'Université:
Publications, Tome 1.
(*Observatory.*) 8vo. Poznań, 1931

Pračka (Ladislav):
Untersuchungen über den Lichtwechsel älterer veränderlicher Sterne nach den Beobachtungen von Prof. Dr. Vojtěch Šafařík in Prag. Vol. II.
(*Institut astronomique de l'Université Charles.*) Fol. Prag, 1916
Vol. 2. Sterne des A. G. Kataloges von 5^h 21^m bis 24^h A. R.

Prague, Observatoire des Frères Frič à Ondřejov:
Publications, Číslo 1.
(*Observatory.*) 8vo. Prague, 1929

Proctor (R. A.):
Half Hours with the Stars. A plain and easy guide to the knowledge of the constellations. New Impression.
(*Publishers.*) 4to. London, 1921

Pulkovo Observatory:
Refraction Tables of Pulkovo Observatory. Third edition.
(*Observatory.*) 8vo. Moscow, 1930

Pulkovo Observatory:
Comité de l'Heure, Bulletin, Nos. 10–14.
(*Observatory.*) 8vo. Leningrad, 1931

Rio de Janeiro, Observatorio Nacional:
——: Boletim Sismologico do Observatorio Nacional 1926 a 1929.
(*Observatory.*) 8vo. Rio de Janeiro, 1930
——: Taboas das Mares para o anno de 1931. . . .
(*Observatory.*) 8vo. Rio de Janeiro, 1930

Rodés (Luis):
Una nueva determinación de la distancia solar.
(*Tortosa Observatory.*) 8vo. Tortosa, 1929

Rome, Reale Accademia d'Italia:
——: Memorie. Classe di Scienze fisiche, matematiche e naturali. Fisica, Vol. 1, Nos. 1, 2; Matematica, Vol. 1, Nos. 1–5.
(*Academy.*) 8vo. Roma, 1930

Rome, Specola Astronomica Vaticana:
Catalogo Astrografico 1900·0. Sezione Vaticana, Decl. da $+55°$ a $+65°$ . . . calcolate all'Osservatorio di Oxford. Appendice III, pt. 2. Dec. $+58°\ 55'$ a $+53°\ 55'$.
(*Observatory.*) 4to. Roma, 1930

App. III, pt. 2. Catalogo di stelle doppie con distanze inferiori a 15″, contenute nelle zone astrografiche vaticane, cercate e discusse da J. Stein, S.J.

San Fernando, Instituto y Observatorio de Marina:
Carta Fotográfica del Cielo. Zona $-3°$. (10 Charts.)
(*Observatory.*)

St John (C. E.):
Chemistry of the Sun. Periodic Table of the Elements grouped according to the Number of Outer (Valence) Electrons.
(*Author.*) Large sheet map, MS., 1930

Savage (Grace O.):
*Printed Treasures in the Library of the United States Naval Observatory.
(*U.S. Naval Institute.*) 8vo. [Annapolis], [1930]

Schlesinger (Frank):
Catalogue of Bright Stars, containing all important data known in June 1930, relating to all stars brighter than 6·5 visual magnitude, and to some fainter ones. With appendices containing galactic co-ordinates and indices to the constellations.
(*Yale University Observatory.*) 4to. New Haven, 1930

Sengupta (P. C.):
*Greek and Hindu Methods in Spherical Astronomy.
(*Author.*) 8vo. Calcutta, 1931

Shapley (Harlow):
Star Clusters. Harvard Observatory Monographs, No. 2.
(*Purchased.*) 8vo. New York, 1930

Sitter (W. de):
*The Expanding Universe.
(*Author.*) 8vo. Milano, 1931

Smart (W. M.):
Text-book on Spherical Astronomy.
(*Purchased.*) 8vo. Cambridge, 1931

Solà (J. Comas):
La théorie corpusculaire ondulatoire de la lumière.
(*H. Kobold.*) 4to. Kiel, 1930

Stetson (H. T.):
Description of the 69-inch reflector of the Perkins Observatory of the Ohio Wesleyan University. [*The Telescope*, No. 1, 1931.]
(*The Perkins Observatory.*) 8vo. [Ohio], 1931

Stockholm, Direction Générale des Télégraphes de Suède:
Mémoires. Étude des courants telluriques, par David Stenquist. Deuxième fascicule.
(*Stockholm, Kungl. Telegrafstyrelsen.*) 4to. Stockholm, 1930

Stracke (G.):
Bahnbestimmung der Planeten und Kometen.
(*Purchased.*) 8vo. Berlin, 1929

Stroobant (P.):
Les observatoires astronomiques et les astronomes. Ouvrage publié sous les auspices de l'Union Astronomique Internationale.
(*I.A.U.*) 8vo. Tournai, Paris, 1931

Swarthmore College, Sproul Observatory:
Publications. No. 11.
(*Observatory.*) 8vo. Philadelphia, 1930
No. 11. Winthrop R. Wright and Heber D. Curtis, Interference in the Solar Corona.

Swings (P.):
——: *Sur la structure des groupes de raies de résonance de la vapeur de soufre.
(*Chief of the Geographical Section, General Staff, War Office.*) 8vo. Cracow, 1929
——: *Sur le spectre de résonance de la vapeur de soufre.
(*Chief of the Geographical Section, General Staff, War Office.*) 8vo. Varsovie, 1930

Sydney Observatory:
Astrographic Catalogue, 1900·0. Sydney Section, Dec. $-51°$ to $-65°$. Vols. 11, 12.
(*Observatory.*) 4to. Sydney, 1930

Vol. 11. R.A. 12^h to 18^h. Dec. 53° to 55°.
Vol. 12. R.A. 18^h to 24^h. Dec. 53° to 55°.

Tams (E.):
——: *Die Seismizität des Südantillenbogens.
(*Hamburg, Hauptstation für Erdbebenforschung am Physikalischen Staatsinstitut.*) 8vo. Braunschweig, 1930
——: *Das Epizentrum des südatlantischen Grossbebens vom 27 Juni 1929.
(*Hamburg, Hauptstation für Erdbebenforschung am Physikalischen Staatsinstitut.*) 8vo. Braunschweig, 1930

Thomas (Oswald):
The Heavens and the Universe. Translated by Bernard Miall.
(*Publishers.*) 8vo. London, 1930

Valkenburg, Sternwarte des Ignatiuskollegs:
Veröffentlichungen. No. 1.
(*Observatory.*) 4to. Valkenburg, 1930

Webb (R. Stretton):
*Notes on the History of Geodesy.
(*Author.*) 8vo. [Cape Town], 1930

Willis (John E.):
Bulletins on Astronomical Practice. Nos. 1–3.
(*Author.*) Sm. 8vo. Washington, 1930

Wood (H. E.):
Recent Astronomical Developments. Presidential Address delivered 1930 July 7. [South African Journal of Science, Vol. XXVII.]
(*Association.*) 8vo. Johannesburg, 1930

Zinner (Ernst):
——: *Das Wetter von Bamberg.
——: *Alte Sonnenuhren an Bamberger Gebäuden.
(*Bamberg Observatory.*) 8vo. Bamberg, 1929
——: *Astronomiegeschichtliche Forschungen.
(*Bamberg Observatory.*) 8vo. Roma, 1930
——: *Die Geschichtschreibung der Naturwissenschaften.
(*Bamberg Observatory.*) 4to. Berlin, 1930
——: Die Geschichte der Sternkunde von den ersten Anfängen bis zur Gegenwart.
(*Purchased.*) 8vo. Berlin, 1931

PHOTOGRAPHS, Etc., PRESENTED TO THE SOCIETY.

Dr. P. Fox. Eleven prints of the Adler Planetarium and Museum, Chicago.

Mr. J. J. Hall. Gold pocket chronometer (Padbury 3474) in mahogany case.

Miss F. Herschel. Portrait of Miss Francisca Herschel taken by Sir W. J. Herschel, Bart., by the process of Lumière's Colour Photography in 1916.

The Warner & Swasey Company. Bronze Medal in commemoration of the 50th anniversary of the Warner & Swasey Company, 1880–1930.

INDEX

PAGE

Annual General Meeting, Report of the proceedings of the..... 299
Auditors of the Treasurer's accounts for 1930, Appointment of the..... 212
—— —— —— —

PAGE

Cowling, T. G., On a point-source model of a star 92
———————— Note on the fitting of polytropic models in the theory of stellar structure 472
Crommelin, A. C. D., Presidential address delivered by, on the award of the Gold Medal to Professor W. de Sitter 422
———————— ———————— On the award of the Jackson-Gwilt Medal and gift to Mr. Clyde W. Tombaugh 434

Dufay, J., Effect of atmospheric absorption in stellar spectrophotometry 549
Dugan, R. S., H. N. Russell and, Apsidal motion in Y Cygni and other stars 212
Dunsink Observatory, Report of the proceedings of 348
Durham University Observatory, Report of the proceedings of the 348
Dynamical Astronomy: Expansion of the disturbing function in terms of the true longitude of the disturbed planet, C. A. Shook 553

Eddington, Sir A. S., The opacity of extended stellar envelopes 109
———————— A theorem concerning incomplete polytropes 440
———————— Upper limits to the central temperature and density of a star 444
Edinburgh, Royal Observatory, Report of the proceedings of the 341
Election void 301
Elvey, C. T., O. Struve and, Algol and stellar rotation 663
Emden's equation: Numerical integration of Emden's polytropic equation of index three, N. Fairclough 55
———————— The solutions of Emden's and similar differential equations, R. H. Fowler 63
———— ———————— On Emden's differential equation, E. Hopf 653
———————— Note on steady-state distributions which are given by solutions of Emden's differential equation, E. A. Milne 751
Errata 256, 606, 703, 949, 1009
Espin, Rev. T. E., Report of his observatory (Wolsingham) 368
———————— and W. Milburn, New double stars 294
Evershed, J., The shift towards red of the calcium, aluminium, and iron lines in the solar spectrum 260
Expansion of the universe, The problem of n bodies and the, G. C. McVittie 274
———— — — ———— — — Council note on 412
———————— A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulæ, G. Lemaître 483
—— ———————— The expanding universe, G. Lemaître 490
———————— ———— De Sitter's universe and the motion of nebulæ, G. Castelnuovo 829

Fairclough, N., Numerical integration of Emden's polytropic equation of index three 55
Fellows elected 1, 211, 257, 300, 439, 607, 705, 821
Ferraro, V. C. A., A note on the possible emission of electric currents from the sun 174
———————— On recombination in ionized streams of corpuscles from the sun 184
Fletcher, A., Note on astronomical refraction 559
———————— ———————— Erratum 1009
Fotheringham, J. K., Note on the motion of the perihelion of Mercury ... 1001
Fowler, R. H., The solutions of Emden's and similar differential equations 63

Galactic Rotation: Star-streaming in galactic zones, R. D. H. Jones 563
George Darwin Lecture delivered by Professor W. de Sitter 706
George Darwin Lecturer for 1931, Professor W. de Sitter appointed 258
Gerasimovič, B. P., Luminosity curve of early B stars 537
Gifts to the general funds of the Society 311

PAGE

Gold Medal of the Society awarded to Professor W. de Sitter, for his theoretical investigations on the orbits of the satellites of Jupiter and his contributions to the theory of relativity.....258, 422
Goryatscheff, N., Occultations of stars by the moon, observed at Tomsk University Observatory during the year 1930..... 604
Greaves, W. M. H., The effect of atmospheric absorption in stellar spectrophotometry..... 938
Greenwich, Royal Observatory, Stellar parallaxes..... 200
——————— A solar eruption on 1930 November 25 .. 239
——————— Mean areas and heliographic latitudes of sunspots in the year 1929..... 271
——————— Report of the proceedings of the..... 339
——————— Observations of solar flocculi made with the spectrohelioscope during the year 1930..... 593
——————— Preliminary values of the variation of latitude at Greenwich in 1930..... 601
——————— Mean areas and heliographic latitudes of sunspots in the year 1930..... 1004
Grove-Hills Special Fund..... 312

Hargreaves, F. J., Report of his observatory (Kingswood)..... 368
Helwan Observatory, Egypt, Report of the proceedings of..... 360
Hindle, J. H., A new test for Cassegrainian and Gregorian secondary mirrors..... 592
Hopf, E., On Emden's differential equation..... 653
Hyderabad, Nizamiah Observatory, Report of the proceedings of the.... 361

Instruments and Appliances, etc.: A new photo-electric recording microphotometer, J. A. Carroll and E. B. Moss..... 191
——————— A new method of second difference integration on the Brunsviga-Dupla machine, E. Chappell..... 817
——————— Note on Mr. Chappell's method of second difference integration, L. J. Comrie..... 819
Instruments belonging to the Society, Report on the..... 310
Investments..... 308

Jackson, J., and W. Bowyer, The Shortt clocks at Greenwich in 1930.... 291
Jackson-Gwilt Medal and Gift, awarded to Mr. Clyde W. Tombaugh for his discovery of the trans-Neptunian planet Pluto.....258, 434
Jeffreys, Harold, Convection in stars..... 121
——————— The resonance theory of the origin of the moon (second paper)..... 169
Johannesburg, Union Observatory, Report of the proceedings of the..... 363
Jones, H. Spencer, The parallax of Nova Pictoris..... 608
——————— The spectrum of Nova Pictoris, 1926–1931..... 777
——————— Note on the spectrum of η Carinæ..... 794
Jones, R. D. H., Star-streaming in galactic zones..... 563
——————— Space distribution of the Boss stars..... 990
Jupiter: Jupiter's Galilean satellites (George Darwin Lecture), W. de Sitter 706
——— Rapidly moving spots on Jupiter's north temperate belt, B. M. Peek..... 941

Knox-Shaw, H., and H. G. Scott Barrett, Faint stars in the Selected Areas with large proper motion. Revised List from the Radcliffe Observatory, Oxford..... 933
Kodaikánal Observatory, Report of the proceedings of..... 362
Kreiken, E. A., On the relation between colour and spectral type in the different galactic latitudes..... 242
——————— Note on the axial rotation of the stars..... 251
——————— Some further remarks on the rotation of the stars..... 501
——————— Further remarks on stellar rotation..... 756

PAGE
Latitude variation, Preliminary values of, at Greenwich in 1930..... 601
Lemaître, G., A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulæ 483
———— The expanding universe..... 490
———— ———— Errata..... 703
Library, Regulation VII, Alteration of..... 299
——— List of donors to the..... 420
——— List of additions to the..... 1011
Liverpool Observatory and Tidal Institute, Report of the proceedings of the..... 349
Lockyer, W. J. S., Two new bright-hydrogen-line B5-type stars..... 187
——— ———— The spectrum of H.D. 20336 in Camelopardalis (Sp. Type B3*nep*) throughout a complete cycle (1925–1930)..... 215
———— A new bright-hydrogen-line star of spectral type B8 in Lacerta..... 259
———— On the relationship between solar prominences and the forms of the corona..... 797
London, University of, and University College Observatories, Report of the proceedings of..... 350
———— Occultations of stars by the moon, observed during the year 1930..... 949
Loomis, A. L., The precise measurement of time..... 569
Luyten, W. J., Note on Dr. Kreiken's paper: "On the dwarf nature of spectroscopic binaries"..... 938

McCrea, W. H., Model stellar atmospheres..... 836
———— and G. C. McVittie, On the contraction of the universe.. 128
McVittie, G. C., The problem of *n* bodies and the expansion of the universe..... 274
———— W. H. McCrea and, On the contraction of the universe.. 128
Melbourne Observatory, Report of the proceedings of..... 364
Melotte, P. J., The numbers of stars per square degree, magnitude 11 to 14, in different galactic latitudes..... 810
Menzel, D. H., Pressures at the base of the chromosphere: a critical study of Milne's theories..... 628
Mercury: Photographies de la planète Mercure, F. Quénisset..... 947
——— Note on the motion of the perihelion of Mercury, J. K. Fotheringham..... 1001
Meteors: On the observation of faint meteors, as experienced in the case of those from the orbit of comet Schwassmann-Wachmann, 1930 *d*, K. Nakamura..... 204
——— ——— Note on the above paper, I. Yamamoto..... 209
Milburn, W., Rev. T. E. Espin and, New double stars..... 294
Miller, J. C. P., The effect of opacity in the point-source stellar model... 984
Milne, E. A., The analysis of stellar structure..... 4
———— ———— Errata..... 256
———— Note on "equations of fit" in the theory of stellar structure 479
———— ———— Errata..... 703, 1009
———— Note on steady-state distributions which are given by solutions of Emden's differential equation..... 751
Minnaert, M., and G. F. W. Mulders, Note on Prof. Plaskett's paper: "The formation of the magnesium *b* lines in the solar atmosphere"... 1007
Minor Planets, Council note on the discovery of, in 1930..... 373
———— Observations of, and of comet Nagata (1931 *b*), made with the 243 mm. Merz-Repsold refractor at the Astronomical Observatory, Kiew, S. D. Tscherny..... 1008
Mirrors, A new test for Cassegrainian and Gregorian secondary, J. H. Hindle..... 592
——— Note on the Hartmann test for parabolic and hyperbolic, R. A. Sampson..... 862

PAGE

Moon, Origin of the, The resonance theory of the (Second paper), Harold Jeffreys 169
Moss, E. B., J. A. Carroll and, A new photo-electric recording microphotometer 191
Mulders, G. F. W., M. Minnaert and, Note on Prof. Plaskett's paper: "The formation of the magnesium *b* lines in the solar atmosphere" 1007

Nakamura, K., On the observation of faint meteors, as experienced in the case of those from the orbit of comet Schwassmann-Wachmann, 1930 *d* 204
——————— ——————— Note on the above paper, I. Yamamoto ... 209
Nebulæ, Council note on 416
——— A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulæ, G. Lemaître 483
——— De Sitter's universe and the motion of nebulæ, G. Castelnuovo 829
Newbegin, A. M., Report of his observatory (Worthing) 369

Obituary:—
List of Fellows and Associates deceased 315
Obituary Notice: Associate:—
Hagen, Johann Georg 337
Obituary Notices: Fellows:—
Adams, Harold John 315
Bird, John Turnbull 315
Bulkeley, Richard Henry 316
Eddie, Charles 316
Gifford, J. William 317
Knobel, Edward Ball 318
Tondorf, Francis Anthony 321
Turner, Herbert Hall 321
Wilson, William 334
Wood, Hugh Montague 335
Young, Alfred Ernest 336
Observatories, Work at various: see reports of individual observatories (or observers) under their respective headings.
Occultations of stars by the moon, observed at the University of Birmingham during the year 1930, J. Young 604
——————————— observed at Tomsk University Observatory during the year 1930, N. Goryatscheff 604
——————————— observed during the year 1930, University of London Observatory 949
Officers and Council, List of, 1931–32 438
Ottawa, Dominion Observatory, Report of the proceedings of the 356
Oxford, Radcliffe Observatory, Report of the proceedings of the 352
——————————— Faint stars in the Selected Areas with large proper motion. Revised list, H. Knox-Shaw and H. G. Scott Barrett 933
Oxford University Observatory, Report of the proceedings of the 352

Pannekoek, A., The theoretical contours of absorption lines 139
——————— ——————— (Second paper) 519
Peek, B. M., Photometric observations of Nova Persei, 1901 203
——————— Report of his observatory (Solihull) 369
——————— Rapidly moving spots on Jupiter's north temperate belt 941
Phillips, Rev. T. E. R., Micrometrical measures of double stars 238
——————————— Report of his observatory (Headley, Epsom) 370
Photography, Celestial: List of reproductions of celestial photographs by the Society 314, 440

PAGE

Pickering, W. H., The discovery of Pluto 812
——————— ——————— Errata 1009
Plakidis, S., Observations of Comet Forbes, 1930 *e* 210
Plaskett, H. H., The formation of the magnesium *b* lines in the solar atmosphere 870
——————— ——————— Note on the above paper, M. Minnaert and G. F. W. Mulders 1007
Pluto, Note on the revision of the measure of the image of, on the Uccle plate of 1927 January 27 210
——— Jackson-Gwilt Medal and Gift awarded to Mr. Clyde W. Tombaugh for his discovery of 258, 434
——— Council note on the discovery of 380
——— The discovery of, W. H. Pickering 812
——— ——————— Errata 1009
Presents announced 3, 212, 258, 440, 608, 706, 821
Progress and present state of the Society 301
Publications of the Society 314

Quénisset, F., Photographies de la planète Mercure 947

Refraction, Note on astronomical, A. Fletcher 559
——————— ——————— Erratum 1009
Relativity: On the contraction of the universe, W. H. McCrea and G. C. McVittie 128
——————— The shift towards red of the calcium, aluminium, and iron lines in the solar spectrum, J. Evershed 260
——————— The problem of *n* bodies and the expansion of the universe, G. C. McVittie 274
——————— Council note on the expansion of the universe 412
——————— The dissociation formula according to the relativistic statistics, S. Chandrasekhar 446
——————— The highly collapsed configurations of a stellar mass, S. Chandrasekhar 456
——————— ——————— Erratum 949
——————— A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extra-galactic nebulæ, G. Lemaître 483
——————— The expanding universe, G. Lemaître 490
——————— De Sitter's universe and the motion of nebulæ, G. Castelnuovo 829
——————— Note on the motion of the perihelion of Mercury, J. K. Fotheringham 1001
Russell, H. N., Composite polytropic gas spheres 739
——————— Notes on the constitution of the stars 951
——————— and R. S. Dugan, Apsidal motion in Y Cygni and other stars 212

Sampson, R. A., Note on the Hartmann test for parabolic and hyperbolic mirrors 862
Sargent, F., Variability of B.D. +72°733 602
Shajn, G., On the behaviour of certain simple multiplets in stellar spectra 675
Shook, C. A., Expansion of the disturbing function in terms of the true longitude of the disturbed planet 553
Shortt clocks, The, at Greenwich in 1930, J. Jackson and W. Bowyer 291
——————— The precise measurement of time, A. L. Loomis 569
——————— Analysis of records made on the Loomis chronograph by three Shortt clocks and a crystal oscillator, E. W. Brown and D. Brouwer 575
Sidmouth, Norman Lockyer Observatory, Report of the proceedings of the 354

PAGE

Sitter, W. de, awarded the Gold Medal for his theoretical investigations on the orbits of the satellites of Jupiter and his contributions to the theory of relativity....258, 422
———— appointed George Darwin Lecturer for 1931.... 258
———— Jupiter's Galilean satellites (George Darwin Lecture).... 706
———— ———— Erratum.... 949
South Kensington, Imperial College Observatory, Report of the proceedings of the.... 348
Spectro-photometry: Effect of atmospheric absorption in stellar spectrophotometry, J. Dufay.... 549
———— The effect of atmospheric absorption in stellar spectro-photometry, W. M. H. Greaves.... 938
Spectroscopy, Astronomical: Wave-lengths of oxygen and nitrogen lines in the stellar region, C. S. Beals.... 531
———— Interlocking of triplets of absorption lines, R. v. d. R. Woolley.... 864
———— The contours of emission bands in novæ and Wolf-Rayet stars, C. S. Beals.... 966
———— Selective absorption, R. v. d. R. Woolley.... 977
Spherical Astronomy: Graphical solution of a spherical triangle, L. Becker 226
Star Colours: On the relation between colour and spectral type in the different galactic latitudes, E. A. Kreiken.... 242
Star Temperatures: Effect of atmospheric absorption in stellar spectrophotometry, J. Dufay.... 549
———— ———— W. M. H. Greaves.... 938
Stars, Axial Rotation of: Note on the axial rotation of the stars, E. A. Kreiken.... 251
———— Some further remarks on the rotation of the stars, E. A. Kreiken.... 501
———— Algol and stellar rotation, O. Struve and C. T. Elvey.... 663
———— Further remarks on stellar rotation, E. A. Kreiken.... 757
Stars, Constitution and Evolution of: The analysis of stellar structure, E. A. Milne.... 4
———— ———— Errata.... 256
———— Numerical integration of Emden's polytropic equation, N. Fairclough.... 55
———— The solutions of Emden's and similar differential equations, R. H. Fowler.... 63
———— On a point-source model of a star, T. G. Cowling.... 92
———— The opacity of extended stellar envelopes, Sir A. S. Eddington.... 109
———— Convection in stars, Harold Jeffreys 121
———— The slow contraction or expansion of a fluid sphere. I. Equilibrium, L. H. Thomas.... 122
———— Note on the temperature distribution in the deep layers of stellar atmospheres, M. Bronstein.... 133
———— The theoretical contours of absorption lines, A. Pannekoek.... 139
———— The transmutation of elements in stars, A. H. Wilson.... 283
———— A theorem concerning incomplete polytropes, Sir A. S. Eddington.... 440
———— Upper limits to the central temperature and density of a star, Sir A. S. Eddington.... 444
———— The dissociation formula according to the relativistic statistics, S. Chandrasekhar.... 446
———— The highly collapsed configurations of a stellar mass, S. Chandrasekhar.... 456

PAGE

Stars, Constitution and Evolution of: The possible solutions of the "equations of fit" on the standard model, B. Strömgren..... 466
——— Note on the fitting of polytropic models in the theory of stellar structure, T. G. Cowling..... 472
——— Note on "equations of fit" in the theory of stellar structure, E. A. Milne..... 479
——— Errata 703, 1009
——— The theoretical contours of absorption lines. (Second paper), A. Pannekoek..... 519
——— The slow contraction or expansion of a fluid sphere. II. Stability, L. H. Thomas..... 619
——— On Emden's differential equation, E. Hopf..... 653
——— Composite polytropic gas spheres, H. N. Russell..... 739
——— Note on steady-state distributions which are given by solutions of Emden's differential equation, E. A. Milne..... 751
——— Model stellar atmospheres, W. H. McCrea..... 836
——— The absorption coefficient of a degenerate gas, Bertha Swirles..... 857
——— Notes on the constitution of the stars, H. N. Russell..... 951
——— The effect of opacity in the point-source stellar model, J. C. P. Miller..... 984

Stars, Distribution and Motions of: Star-streaming in galactic zones, R. D. H. Jones..... 563
——— Space distribution of the Boss stars, R. D. H. Jones..... 990

Stars, Distribution and Statistics of: On the relation between colour and spectral type in the different galactic latitudes, E. A. Kreiken.. 242
——— Luminosity curve of early B stars, B. P. Gerasimovič..... 537
——— The numbers of stars per square degree, magnitude 11 to 14, in different galactic latitudes, P. J. Melotte..... 810

Stars, Double and Multiple: Micrometrical measures of double stars, Rev. T. E. R. Phillips..... 238
——— New double stars, Rev. T. E. Espin and W. Milburn..... 294
——— Council note on..... 405
——— Note on Dr. Kreiken's paper: "On the dwarf nature of spectroscopic binaries," W. J. Luyten..... 938

Stars, Magnitudes of: Luminosity curve of early B stars, B. P. Gerasimovič..... 537
——— Note on Dr. Kreiken's paper: "On the dwarf nature of spectroscopic binaries," W. J. Luyten..... 938

Stars, New: Photometric observations of Nova Persei, 1901, B. M. Peek.. 203
——— The parallax of Nova Pictoris, H. Spencer Jones..... 608
——— The spectrum of Nova Pictoris, 1926–1931, H. Spencer Jones 777
——— Observations of Nova Persei (1901), 1930–1931, W. H. Steavenson..... 941

Stars, Parallaxes of: Stellar parallaxes, Royal Observatory, Greenwich... 200
——— The parallax of Nova Pictoris, H. Spencer Jones... 608
——— Stellar parallaxes determined photographically at the Cape Observatory. (Third List), Royal Observatory, Cape of Good Hope..... 822

Stars, Proper Motions of: Star-streaming in galactic zones, R. D. H. Jones 563

Stars, Spectra of: Two new bright-hydrogen-line B5-type stars, W. J. S. Lockyer..... 187

PAGE

Stars, Spectra of: The spectrum of H.D. 20336 in Camelopardalis (Sp. Type B3*nep*) throughout a complete cycle (1925–1930), W. J. S. Lockyer..... 215
——————— On the relation between colour and spectral type in the different galactic latitudes, E. A. Kreiken..... 242
——————— A new bright-hydrogen-line star of spectral type B8 in Lacerta, W. J. S. Lockyer..... 259
——————— Council note on stellar spectroscopy in 1930..... 396
——————— On the behaviour of certain simple multiplets in stellar spectra, G. Shajn..... 675
——————— The spectrum of Nova Pictoris, 1926–1931, H. Spencer Jones..... 777
——————— Note on the spectrum of η Carinæ, H. Spencer Jones.. 794
——————— The contours of emission bands in novæ and Wolf-Rayet stars, C. S. Beals..... 966
Stars, Variable: Apsidal motion of Y Cygni and other stars, H. N. Russell and R. S. Dugan..... 212
——————— Discussion of observations of three long-period variables, made by A. N. Brown, M.A. No. 3, U Persei, 1909 September–1930 March, Mary A. Blagg..... 231
——————— Council note on..... 408
——————— Variability of B.D. +72°733, F. Sargent 602
——————— Algol and stellar rotation, O. Struve and C. T. Elvey.... 663
——————— A new variable star in Auriga, A. Stanley Williams..... 698
Steavenson, W. H., Report of his observatory (West Norwood)..... 370
——————— Observations of Nova Persei (1901), 1930–1931..... 941
Stonyhurst College Observatory, Report of the proceedings of..... 355
Strömgren, B., The possible solutions of the "equations of fit" on the standard model..... 466
——————— Erratum..... 703
Struve, O., and C. T. Elvey, Algol and stellar rotation..... 663
Sun: Council note on solar research in 1930..... 387
——————— the solar corona without eclipse..... 395
Sun, Chromosphere and Prominences: A solar eruption on 1930 November 25, Royal Observatory, Greenwich..... 239
——————— Council note on solar activity in 1930..... 386
——————— Pressures at the base of the chromosphere: a critical study of Milne's theories, D. H. Menzel 628
——————— On the relationship between solar prominences and the forms of the corona, W. J. S. Lockyer..... 797
Sun, Corpuscular Radiation from: A note on the possible emission of electric currents from the sun, V. C. A. Ferraro..... 174
——————— On recombination in ionized streams of corpuscles from the sun, V. C. A. Ferraro..... 184
Sun, Eclipses of the: On the relationship between solar prominences and the forms of the corona, W. J. S. Lockyer..... 797
Sun, Spectrum of: The shift towards red of the calcium, aluminium, and iron lines in the solar spectrum, J. Evershed..... 260
——————— Council note on the solar corona without eclipse.... 395
——————— The formation of the magnesium *b* lines in the solar atmosphere, H. H. Plaskett..... 870
——————— Note on the above paper, M. Minnaert and G. F. W. Mulders..... 1007
Sunspots and Faculæ: Observations of solar flocculi made with the spectro-helioscope during the year 1930, Royal Observatory, Greenwich..... 593
——————— Mean areas and heliographic latitudes of sunspots in the year 1929, Royal Observatory, Greenwich..... 271
——————— in the year 1930, Royal Observatory, Greenwich 1004

PAGE
Swirles, Bertha, The absorption coefficient of a degenerate gas 857
Sydney Observatory, Report of the proceedings of 365

Thomas, L. H., The slow contraction or expansion of a fluid sphere. I. Equilibrium 122
——————— ——————— II. Stability 619
Time, The precise measurement of, A. L. Loomis 569
——— Analysis of records made on the Loomis chronograph by three Shortt clocks and a crystal oscillator, E. W. Brown and D. Brouwer 575
Tombaugh, Clyde W., awarded the Jackson-Gwilt medal and gift for his discovery of the extra-Neptunian planet Pluto 258, 434
Tomkins, H. G., Report of his observatory (Dedham) 371
Treasurer's accounts for 1930 302
Trust Funds 312
Tscherny, S. D., Observations of minor planets and of the comet Nagata (1931 *b*), made with the 243 mm. Merz-Repsold refractor at the Astronomical Observatory, Kiew 1008

Victoria, B.C., Dominion Astrophysical Observatory, Report of the proceedings of the 358

Wellington, N.Z., Dominion Observatory, Report of the proceedings of the 365
Williams, A. Stanley, A new variable star in Auriga 698
Wilson, A. H., The transmutation of elements in stars 283
Woolley, R. v. d. R., Interlocking of triplets of absorption lines 864
——————————— Selective Absorption 977

Yamamoto, I., Note on Mr. Nakamura's paper on the observation of faint meteors 209
Young, J., Occultations of stars by the moon, observed at the University of Birmingham during the year 1930 604

University of London
Library.

MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

Vol. XCI. No. 1. November 1930.

PUBLISHED AND SOLD BY THE ROYAL ASTRONOMICAL SOCIETY,
BURLINGTON HOUSE, LONDON, W. 1.

PRICE TO NON-FELLOWS, 4*s.* 0*d*

ANNOUNCEMENT OF PAPERS RECEIVED.

Post-cards, giving the titles of such Papers as have been received or promised up to the Monday preceding the day of the Ordinary Meeting, will be sent to those Fellows who inform the Assistant Secretary of their wish to receive them.

NOTICE TO AUTHORS.

1. Authors are requested to send their manuscripts in as convenient a form for publication as possible. MSS. must be clearly written or typewritten; otherwise they may be returned to the senders to be recast. Tabular matter should be arranged so that it can be printed upright on the page, and the number of columns and the number of figures in each column should be carefully considered from this point of view.

2. Authors should conform as closely as possible to the recommendations of the I.A.U., especially those of Commission 5 on Bibliography (*Trans. I.A.U.*, vol. 3, 1928). In particular, the date (year of publication) of *every* publication referred to, in footnotes or elsewhere, should be given.

3. The unit of wave-length should be written "angstrom" (small a, no accents), *not* "Angstrom Unit." (*Cf.* ampere, volt, henry, etc.) The abbreviated form should be simply "A." (*e.g.* 6563 A.). In spectral classification the decimal divisions should not be written as suffixes, *e.g.* write F5 and *not* F_5.

4. Diagrams or other illustrations accompanying Papers should be sent in a state suitable for direct photographic reproduction. It is preferable that diagrams should be drawn on a larger scale than required, in Indian ink on white paper, all lines, etc., being sufficiently bold to bear the reduction required. Any lettering should be written in pencil, that it may be re-drawn to the proper scale.

CONTENTS.

	PAGE
Fellows elected	1
Candidates proposed	*ib.*
Presents announced	3
E. A. Milne, The Analysis of Stellar Structure	4
N. Fairclough, Numerical Integration of Emden's Polytropic Equation of Index Three	55
R. H. Fowler, The Solutions of Emden's and Similar Differential Equations	63
T. G. Cowling, On a Point-Source Model of a Star	92
Sir Arthur Eddington, The Opacity of Extended Stellar Envelopes	109
Harold Jeffreys, Convection in Stars	121
L. H. Thomas, The Slow Contraction or Expansion of a Fluid Sphere	122
W. H. McCrea and **G. C. McVittie,** On the Contraction of the Universe	128
M. Bronstein, Note on the Temperature Distribution in the Deep Layers of Stellar Atmospheres	133
A. Pannekoek, The Theoretical Contours of Absorption Lines	139
Harold Jeffreys, The Resonance Theory of the Origin of the Moon (Second Paper)	169
V. C. A. Ferraro, A Note on the Possible Emission of Electric Currents from the Sun	174
V. C. A. Ferraro, On Recombination in Ionized Streams of Corpuscles from the Sun	184
W. J. S. Lockyer, Two New Bright-hydrogen-line B5-type Stars	187
J. A. Carroll and **E. B. Moss,** A New Photo-electric Recording Microphotometer	191
Royal Observatory, Greenwich, Stellar Parallaxes	200
B. M. Peek, Photometric Observations of Nova Persei, 1901	203
K. Nakamura, On the Observation of Faint Meteors, as experienced in the case of those from the Orbit of Comet Schwassman-Wachmann, 1930 *d*	204
I. Yamamoto, Note added to the above Paper	209
S. Plakidis, Observations of the Comet Forbes, 1930 *e*	210
Note on the Revision of the Measure of the Image of Pluto on the Uccle Plate of 1927 Jan. 27	*ib.*

Printed in Great Britain by NEILL & Co., LTD., Edinburgh, January 1931.